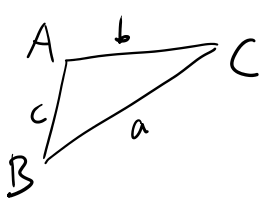
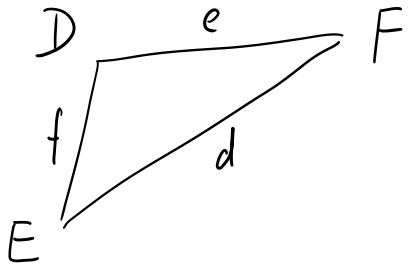
I. Law of Sine

i. Geometry





Axioms of Congruence $\Leftarrow \cong$

SAS (side - angle - side)

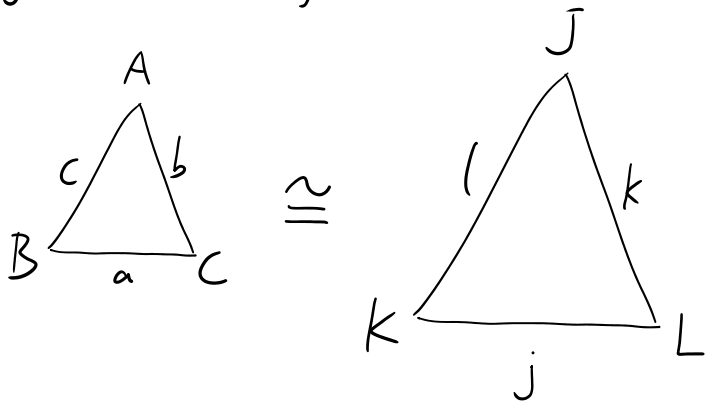
AAS (angle - angle - side)

SSS (side - side - side)

$\triangle_1 \cong \triangle_2$ if any the 3 axioms apply.

It means sides are proportional, and angles are same.

Thus, by the following:



$\Rightarrow \frac{c}{\cdot} = \frac{b}{\cdot} = \frac{a}{\cdot}$

$\Rightarrow \quad \frac{c}{l} = \frac{b}{k} = \frac{a}{j}$

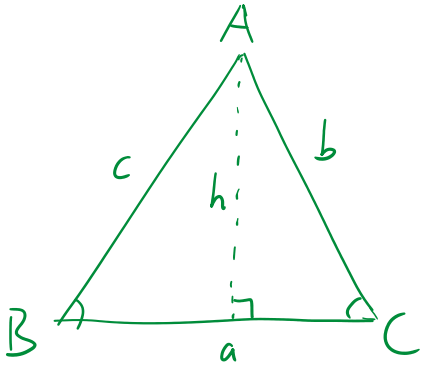
and $\angle A = \angle J$, $\angle B = \angle K$, $\angle C = \angle L$

ii. Law of Sine

For any oblique triangles, ← not ⊾


or


Now,



$\sin B = \frac{h}{c}$, $\sin C = \frac{h}{b}$

$c \cdot \quad c \cdot \qquad b \cdot \quad b \cdot$

$\sin B \cdot c = h$, $\sin C \cdot b = h$ ← equal to h

Set to an equation:

$$\frac{\sin B \cdot c}{b} = \frac{\sin C \cdot b}{b}$$

$$\frac{\frac{\sin B \cdot c}{b}}{\frac{1}{c}} = \frac{\sin C}{c}$$

$\frac{\sin B \cdot c}{b} \cdot \frac{1}{c}$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

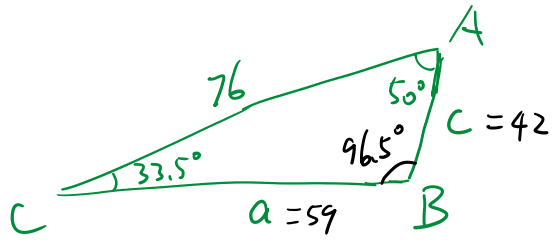
Similarly, we can obtain $\frac{\sin A}{a} = \frac{\sin B}{b}$ and etc.

Therefore, if ΔABC is a non-right triangle, the law of sine is:

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$ or: $$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

eg. Solve ΔABC if $A = 50°$, $C = 33.5°$ and $b = 76$.

Sol:



← draw this would be helpful

$180° - 50° - 33.5° = 96.5°$

$\angle B = 180° - 50° - 33.5° = 96.5°$

By the Law of Sine, we have

$$\frac{\sin 50°}{a} = \frac{\sin 96.5°}{76}$$

$$a \cdot \sin 96.5° = \sin 50° \cdot 76$$

$$\frac{a \cdot \sin 96.5°}{\sin 96.5°} = \frac{\sin 50° \cdot 76}{\sin 96.5°}$$

$$a \approx \boxed{59}$$

,

$$\frac{\sin 33.5°}{c} = \frac{\sin 96.5°}{76}$$

$$c \cdot \sin 96.5° = \sin 33.5° \cdot 76$$

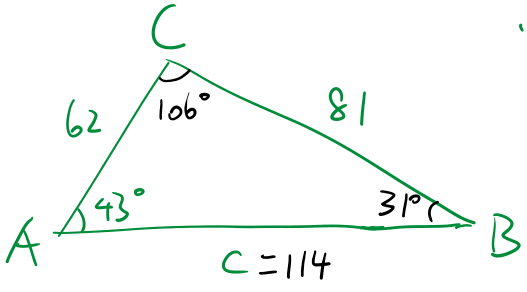
$$\frac{c \cdot \sin 96.5°}{\sin 96.5°} = \frac{\sin 33.5° \cdot 76}{\sin 96.5°}$$

$$c \approx \boxed{42}$$

eg. Solve ΔABC if $A = 43°$, $a = 81$ and $b = 62$.

↑ "20.4" ↑ "20.3"

Sol:



By the Law of Sine:

$$\frac{\sin 43^\circ}{81} = \frac{\sin B}{62}$$

$$\frac{\sin 43^\circ \cdot 62}{81} = \frac{\sin B \cdot \cancel{81}}{\cancel{81}_1}$$

$$0.522 = \sin B$$

$\sin^{-1}$ $\sin^{-1}$

$$\boxed{31^\circ} \approx B$$

$$\angle C = 180^\circ - 43^\circ - 31^\circ = \boxed{106^\circ}$$

$$\frac{\sin 106^\circ}{c} = \frac{\sin 43^\circ}{81}$$

$$\sin 43^\circ \cdot c = \sin 106^\circ \cdot 81$$

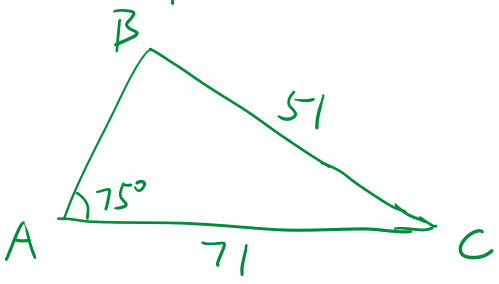
$$c = \frac{\sin 106^\circ \cdot 81}{\sin 43^\circ}$$

$$c \approx \boxed{114}$$

Ambiguous case : ← the Law doesn't alway work

eg. Solve $\triangle ABC$ if $A = 75^\circ$, $a = 51$, and $b = 71$.

sol:



By the Law of Sine

$$\frac{\sin 75^\circ}{51} = \frac{\sin B}{71}$$

$$\frac{\sin B \cdot \cancel{51}}{\cancel{51}} = \frac{\sin 75^\circ \cdot 71}{51}$$

$$\sin B = 1.34 \quad \leftarrow 1.34 > 1$$

$$\boxed{\text{No solution}}$$