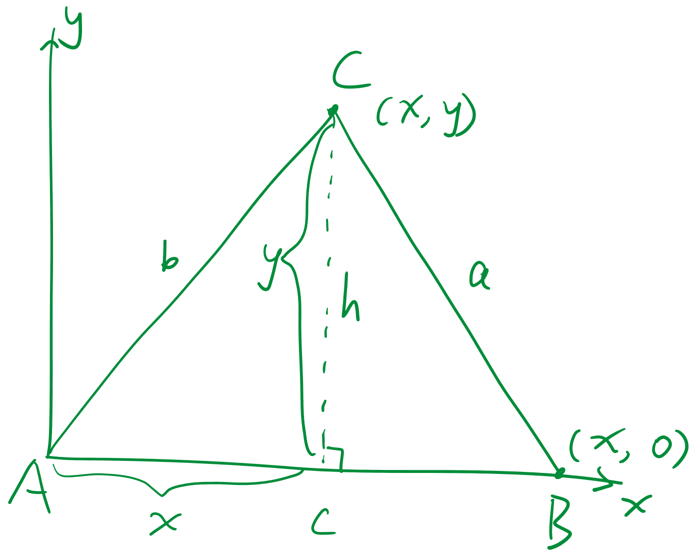
II. Law of Cosine



← Quadrant I

with A from (0, 0)

Then, $\cos A = \frac{x}{b}$, $\sin A = \frac{y}{b}$

$\Rightarrow x = b\cos A$, $y = b\sin A$

$\Rightarrow$ C was (x, y), it is now $(b\cos A, b\sin A)$.

Then, the distance of C to x-axis is (x, y) to $(c, 0)$. (It's y, as well)

We have:

$$a = \sqrt{(x-c)^2 + (y-0)^2}$$

$$(\ \ )^2 \quad (\qquad\qquad)^2$$

$$a^2 = (x-c)^2 + (y-0)^2$$

Subsitute $x = b\cos A$, $y = b\sin A$

$$a^2 = (b\cos A - c)^2 + (b\sin A - 0)^2$$

$$\boxed{\begin{aligned}&(x-y)^2\\&= x^2 - 2xy + y^2\end{aligned}}$$

$$a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A - 2\cdot b\sin A\cdot 0 + 0^2$$

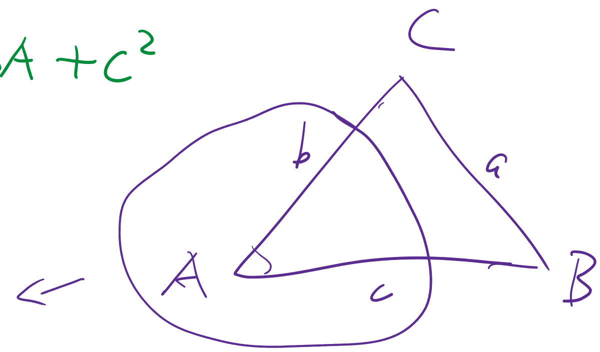
$$a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A - 2\cdot b\sin A\cdot 0 + 0^2$$

$$a^2 = \underline{b^2\cos^2 A} - 2bc\cos A + c^2 + \underline{b^2\sin^2 A}$$

$$a^2 = b^2(\cos^2 A + \sin^2 A) - 2bc\cos A + c^2$$

$$a^2 = b^2\cdot 1 - 2bc\cos A + c^2$$

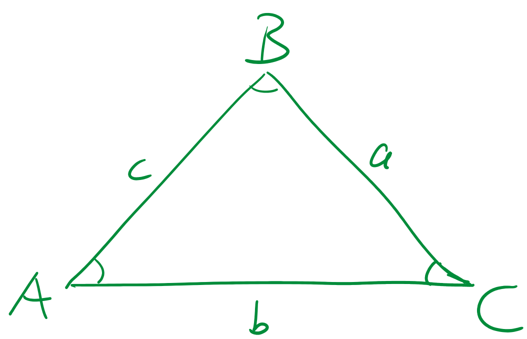
$$a^2 = b^2 + c^2 - 2bc\cos A$$



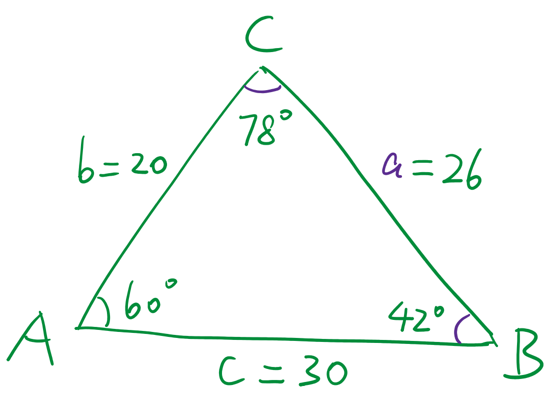
Similarly, we have other two equivalency, in the same manner.

Thus, for $\triangle ABC$ below, with the angles and sides. We have the Law of Cosine:

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$



eg. Solve the triangle below:



Sol: By the Law of Cosine,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 20^2 + 30^2 - 2 \cdot 20 \cdot 30 \cdot \cos 60^\circ$$

$$a^2 = 400 + 900 - 1200 \cdot \frac{1}{2}$$

$$a^2 = 700$$

$$\sqrt{} \qquad \sqrt{}$$

$$a = \pm\sqrt{700} \approx \pm 26$$

$$a = \boxed{26}$$

+ for a side

by the Law of Sine:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

then, $\frac{\sin 60^\circ}{26} = \frac{\sin B}{20}$

$$\frac{\sin 60^\circ \cdot 20}{26} = \frac{\sin B \cdot \cancel{26}^{1}}{\cancel{26}_{1}}$$

$$0.67 = \sin B$$

$$\sin^{-1} 0.67 = B$$

$$\boxed{42^\circ} \approx B$$

$$\angle C = 180^\circ - \angle A - \angle B$$

$$= 180^\circ - 60^\circ - 42^\circ$$

$= \boxed{78°}$

eg. Solve triangle ABC if $a = 6$, $b = 9$ and $c = 4$.

Sol:

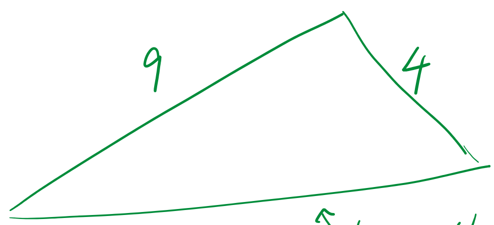
very important

draw it:

$b = 9$, $c = 4$

9 is $4 \cdot 2 + 1$

"more than double



longer than 9

No good!