II. Cont.

eg. Solve triangle ABC if $a = 6$, $b = 9$ and $c = 4$.

Sol:

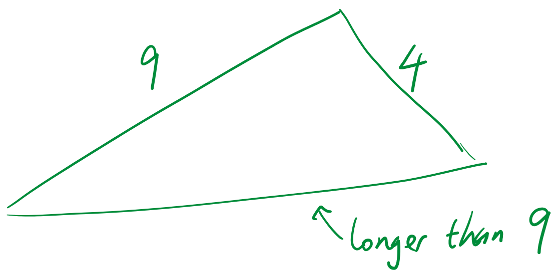
very important

draw it:

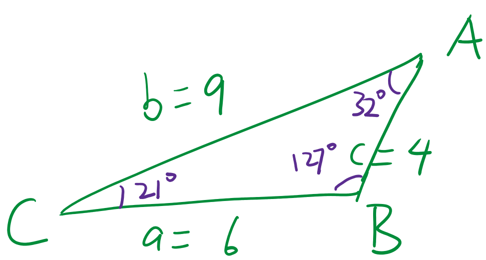
$b = 9$, $c = 4$

9 is $4 \cdot 2 + 1$

"more than double



No good!



no angle

$b^2 = a^2 + c^2 - 2ac\cos B$

Solve for B

$9^2 = 6^2 + 4^2 - 2 \cdot 6 \cdot 4 \cos B$

$81 = 36 + 16 - 48\cos B$

$-36 \quad -36$

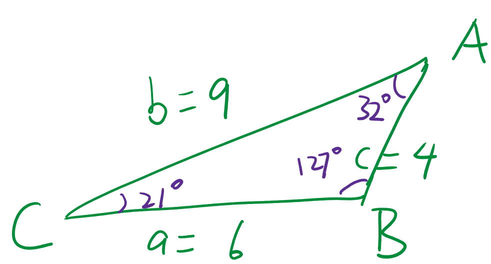
$-16 \quad -16$

$\dfrac{29}{-48} = \dfrac{\cancel{-48}\cos B}{\cancel{-48}}$

$-0.61 = \cos B$

$\cos^{-1} \quad \cos^{-1}$

$\boxed{127°} \approx B$



Now, by the Law of Sine:

$\dfrac{\sin B}{} = \dfrac{\sin A}{}$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 127^\circ}{9} \cdot 6 = \frac{\sin A}{6} \cdot 6$$

$$\frac{\sin 127^\circ}{9} \cdot 6 = \sin A$$

$$0.53 \approx \sin A$$

$$\sin^{-1} \qquad \sin^{-1}$$

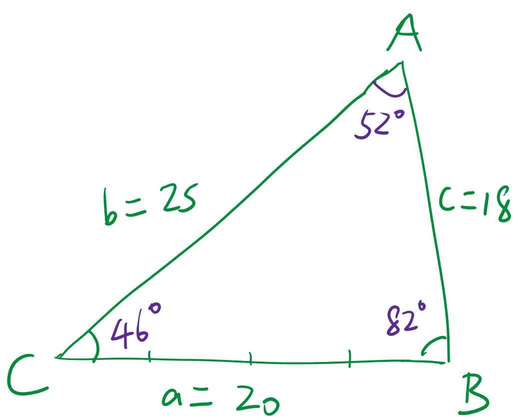
$$\boxed{32^\circ} \approx A$$

$$\angle C = 180^\circ - \angle A - \angle B$$

$$= 180^\circ - 32^\circ - 127^\circ$$

$$= \boxed{21^\circ}$$

Eg. Find the angle α for the given triangle if side $a = 20$, side $b = 25$, and side $c = 18$.



draw it out

$a = 20 \leftarrow 5 \times 4$

$b = 25 \leftarrow 5 \times 5$

$c = 18 \leftarrow 20 - 18 = 2$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$20^2 = 25^2 + 18^2 - 2 \cdot 25 \cdot 18 \cos A$$

$$400 = 625 + 324 - 900\cos A$$

$$400 = 949 - 900\cos A$$

$$-949 \qquad -949$$

$$\frac{-549}{-900} = \frac{-\cancel{900}\cos A}{-\cancel{900}}$$

$$0.61 = \cos A$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 52^\circ}{20} = \frac{\sin B}{25}$$

$$\frac{\sin 52^\circ}{20} \cdot 25 = \sin B$$

$$0.99 = \sin B$$

$$\sin^{-1} \qquad \sin^{-1}$$

$$0.61 = \cos A$$

$\cos^{-1}$ $\cos^{-1}$

$$\boxed{52^\circ} \approx A$$

$\sin^{-1}$ $\sin^{-1}$

$$\boxed{82^\circ} \approx B$$

$$\begin{aligned} \angle C &= 180^\circ - \angle A - \angle B \\ &= 180^\circ - 52^\circ - 82^\circ \\ &= \boxed{46^\circ} \end{aligned}$$

* Heron's Formula for Finding the area.

$$\boxed{\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad \text{where} \quad s = \frac{a+b+c}{2}}$$

This is a great discovery from 60 A.D

Eg. Find the area of the triangle in Figure 9 using Heron's formula.

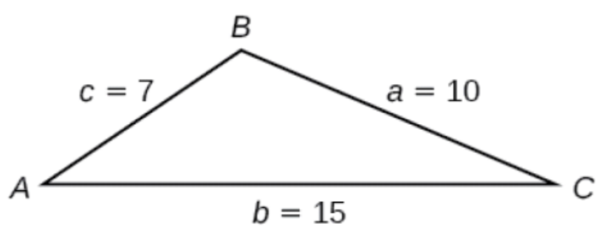


Figure 9

Sol: By the Heron's formula,

$$s = \frac{a+b+c}{2}, \qquad A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{10+15+7}{2} = 16,$$

$$A = \sqrt{16(16-10)(16-15)(16-7)}$$

$$\approx \boxed{29} \text{ unit}^2$$

Eg. Use Heron's formula to find the area of a triangle with sides of lengths $a = 29.7$ ft, $b = 42.3$ ft, and $c = 38.4$ ft.

sol: By the Heron's formula,

$$s = \frac{a+b+c}{2}, \quad A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{29.7+42.3+38.4}{2} = 55.2$$

$$A = \sqrt{55.2(55.2-29.7)(55.2-42.3)(55.2-38.4)}$$

$$\approx \boxed{552.3} \text{ ft}^2$$