II. Arc Length

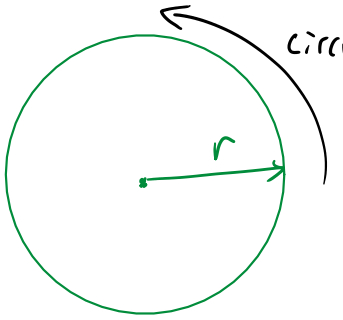
← "thread"; real length of the edge

← "part" of the circumference

The arc length of the circle is the length itself, with respect to the ratio of the radius.

theta

A particular radius will have each arc length.



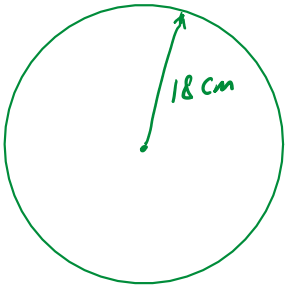
$\Rightarrow$ S is the circumference

θ is the angle (with the radius)

$\Rightarrow$ $\theta \propto \frac{S}{r}$ ← $\frac{\theta}{1} = \frac{S}{r}$

That is, $\boxed{S = r\theta}$, where θ is in radian.

eg. Find the arc length of the circle below.



← full 2π

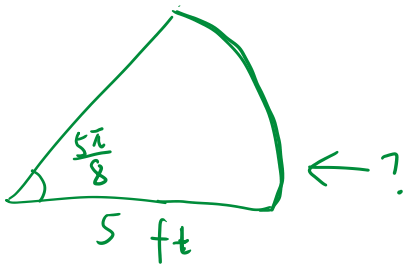
sol: We have $S = r\theta$,

$r = 18$, $\theta = 2\pi$

then, $S = 18 \cdot 2\pi$

$= \boxed{36\pi}$ or $\boxed{113}$ cm

eg. Given the part of the circle below, find its arc length.

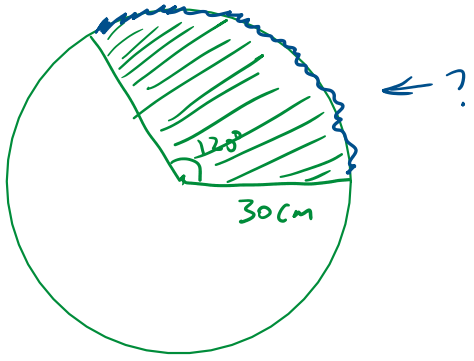


Sol: $s = r\theta$ ← connect to $2\pi r$, then it's no longer $2\pi r$, it is $\theta \cdot r$

$= 5 \cdot \frac{5\pi}{8}$

$\approx \boxed{9.82}$ ft

eg. Find the arclength for the shaded part below:



sol: $s = r\theta$

120° can't $\cdot$ 30 cm

$r = 30$, $120^\circ = 120^\circ \cdot \frac{\pi}{180^\circ} \approx 2.09$

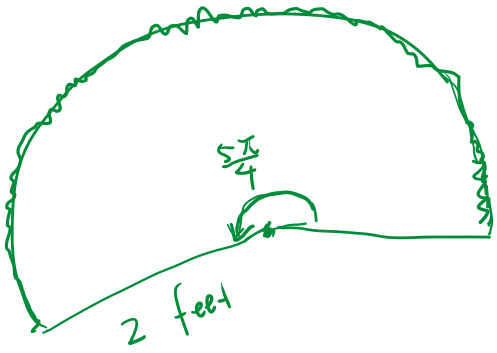
then, $s = r\theta = 30 \cdot 2.09$

180

then, $s = r\theta = 30 \cdot 2.09$

$\approx \boxed{62.7}$ cm

eg. Find the arc length below

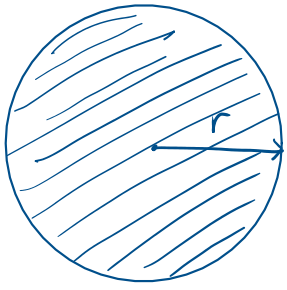


Sol: $s = r\theta$

$= 2 \cdot \frac{5\pi}{4}$

$\approx \boxed{7.85}$ feet

III. Area of a sector ← 'part' of a circle



$A = \pi r^2$

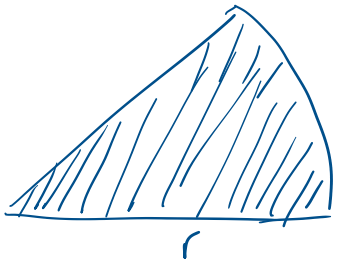
$A = \pi r^2 \cdot 1 = \pi r^2 \cdot \frac{2\pi}{2\pi}$ ← $\pi r^2 \cdot \frac{\theta}{\theta}$

Now, it's for a part: We times a fraction of 2π

Thus,

$A^* = \pi r^2 \cdot \frac{\theta}{\quad}$

intuit.



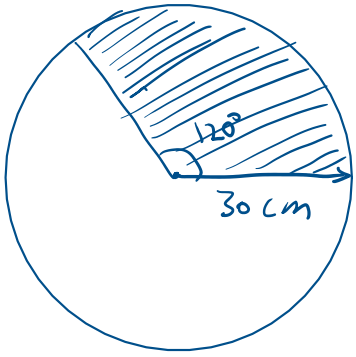
$$A^* = \pi r^2 \cdot \frac{\theta}{2\pi}$$

$$\frac{1}{\pi} r^2 \cdot \frac{\theta}{2\pi}$$

hard to remember

$$\boxed{A = \pi r^2 \frac{\theta}{2\pi}} \quad \text{or} \quad \boxed{A = \frac{1}{2} r^2 \theta}$$

eg. Find the area of the shaded part below:



sol:

$$A = \pi r^2 \cdot \frac{\theta}{2\pi}$$

$$A = \pi \cdot 30^2 \cdot \frac{120^\circ}{360^\circ}$$

$$= \pi \cdot \overset{300}{\cancel{900}} \cdot \frac{\overset{1}{\cancel{120^\circ}}}{\underset{3}{\cancel{360^\circ}}}$$

$$\approx \boxed{942.48} \ cm^2$$

$cm^2 = cm \cdot cm$