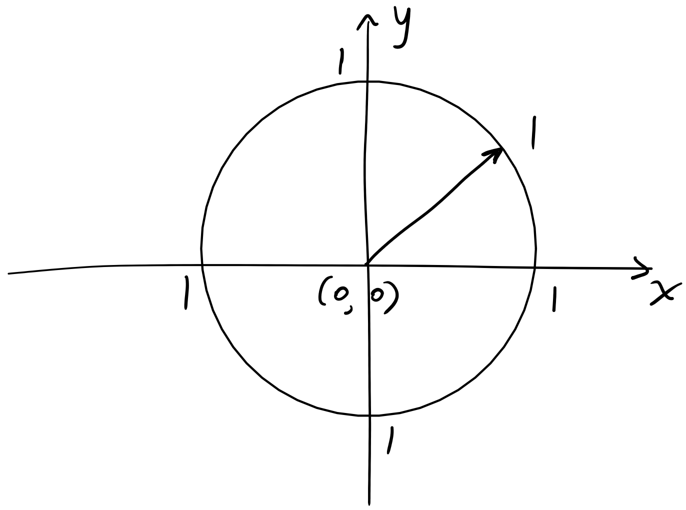
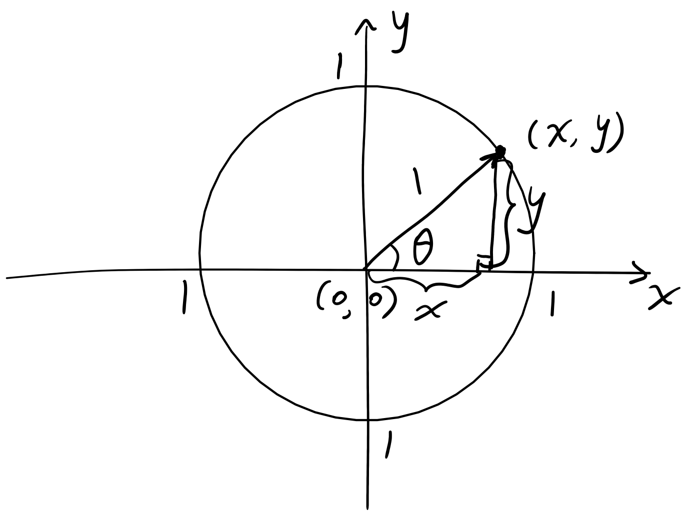
VII. Circle (Circular fct)

For a unit circle on the coordinate system, we have



Then,

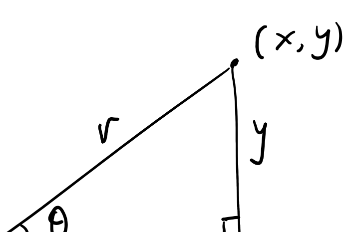


← a point

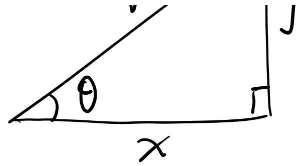
Thus, the point (x, y) is also on the arc the unit circle, and as it is on the coordinate system. That is,

$\boxed{(x, y) = (\cos\theta, \sin\theta)}$, where θ is the angle of the triangle.

Pf:



from above



$$\cos\theta = \frac{x}{r}, \qquad \sin\theta = \frac{y}{r}$$

our radius r is 1, and by the right-triangle:

$$\cos\theta = \frac{x}{1}, \qquad \sin\theta = \frac{y}{1}$$

$$\cos\theta = x, \qquad \sin\theta = y$$

Also, $x^2 + y^2 = r^2$

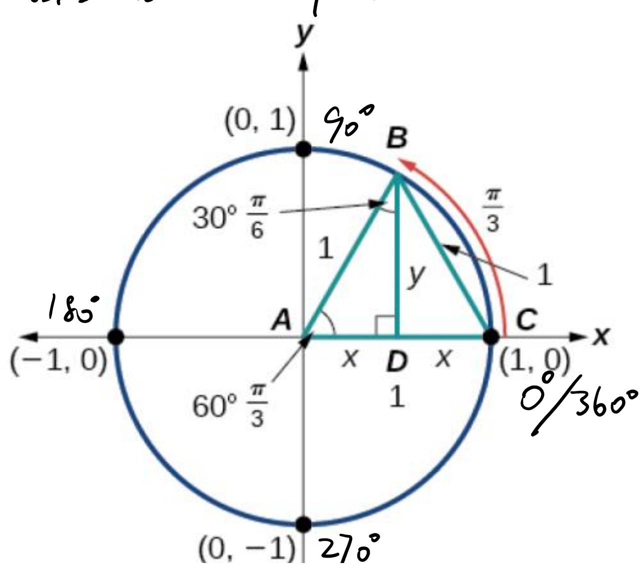
$$\cos^2\theta + \sin^2\theta = 1^2$$

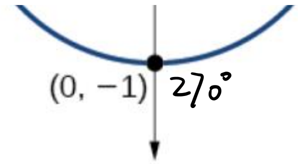
Thus, $\boxed{\cos^2\theta + \sin^2\theta = 1}$

and, $\boxed{\tan\theta = \frac{y}{x}}$ or $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$

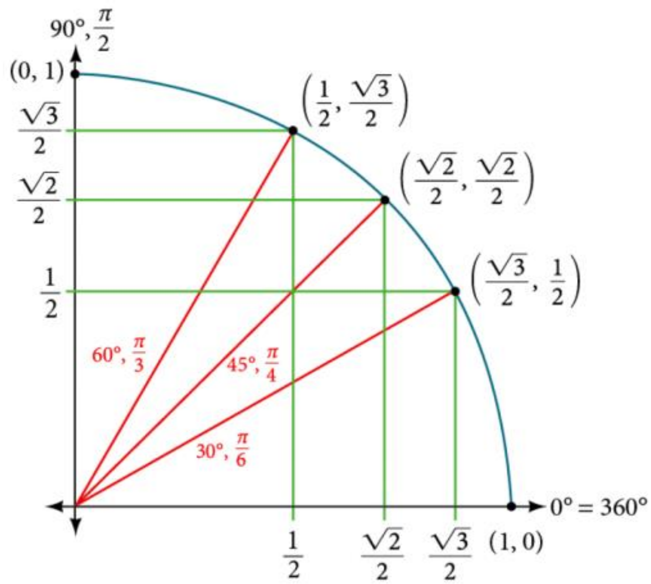
Note: If you draw ⊕, all the "boxed" $\boxed{\cdots}$ will confirm.

From text, this is a reference:



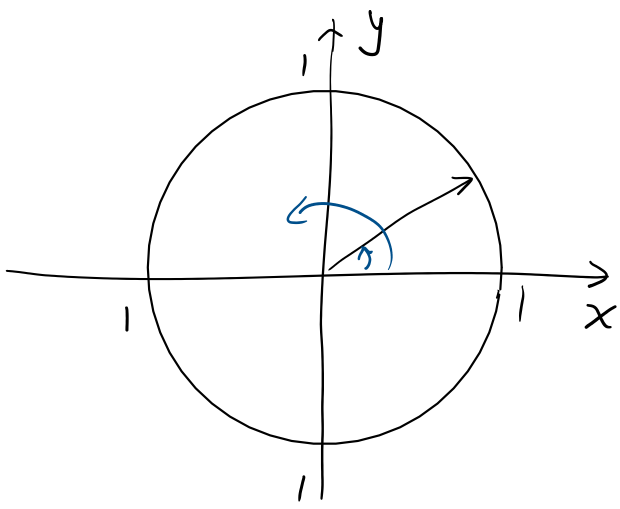


and



← Quadrant I

Now, for $[0, 2\pi]$,



Angle goes to $0° \to 90° \to 180° \to 360°$ (counter clock-wise)

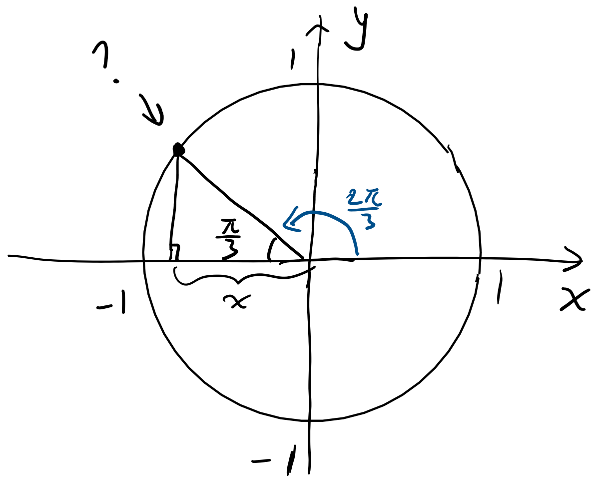
Otherwise, it will be negative.

eg. Evaluate $\cos \frac{2\pi}{3}$.

-1.

eg. Evaluate $\cos\frac{2\pi}{3}$.

Sol: $(x, y) = (\cos\theta, \sin\theta)$



$\frac{2\pi}{3}$ is in QII, because $\frac{\pi}{2}$ is ⌐. Also, $\frac{2\pi}{3} + \frac{\pi}{3} = \frac{3\pi}{3} = \pi$

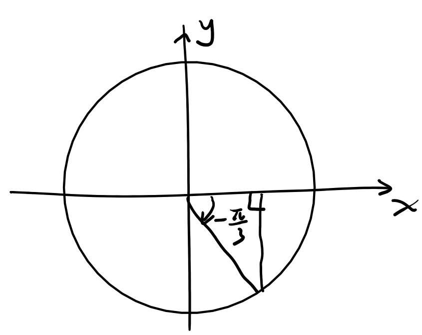
$x = \cos\theta$,

$\cos\frac{2\pi}{3} = \cos\frac{\pi}{3}$ in QII

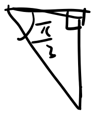
$= \boxed{-\frac{1}{2}}$ ← $-\frac{1}{2}$ because in QII. x is ←

eg. Evaluate $\tan\left(-\frac{\pi}{3}\right)$.

Sol:



$\theta = -\frac{\pi}{3}$, it's clock-wise, ↻



is in QIV

$$\tan\left(-\frac{\pi}{3}\right) = \frac{-y}{x}$$

$$= \boxed{-\sqrt{3}}$$ ← y is negative, x is positive

eg. Evaluate $\sin \frac{19\pi}{4}$.

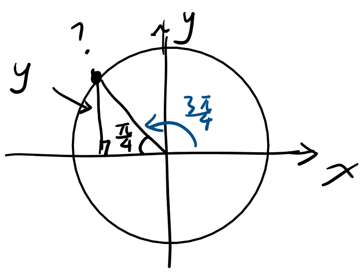
Sol: $\frac{19\pi}{4}$ connects to 2π, 2π with $\frac{19\pi}{4}$, $2\pi = 2\pi \cdot 1 = 2\pi \cdot \frac{4}{4} = \frac{8\pi}{4}$

$$19\pi = 8\pi + 8\pi + 3\pi$$

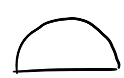
$$\sin \frac{19\pi}{4} = \sin \frac{8\pi + 8\pi + 3\pi}{4}$$

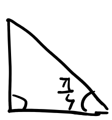
$$= \sin\left(\frac{8\pi}{4} + \frac{8\pi}{4} + \frac{3\pi}{4}\right)$$

$$= \sin \frac{3\pi}{4}$$



← $\frac{3\pi}{4}$ is $\frac{\pi}{4}$ to π



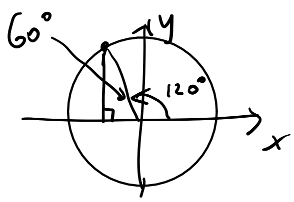


is in QII

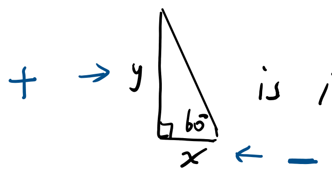
$$\sin \frac{3\pi}{4} = \sin \frac{\pi}{4} \text{ in QII}$$ ← $\sin\theta$ is y

$$= \boxed{\frac{\sqrt{2}}{2}}$$

eg. Evaluate $\tan 120°$.

Sol: 

← 120° is 30° more than 90°



is in QII

$\tan 120° = \tan 60°$ in QII

$= \boxed{-\sqrt{3}}$ ← because $\frac{y}{-x}$