

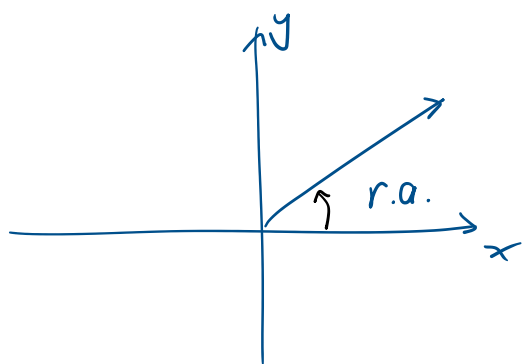
VIII. Reference Angle ← most important

A reference angle is the smaller closest angle to the x-axis.

Book's Defn: It is any non-acute angle that is acute to the x-axis, after the reduction from 180° , 270° , 360° , \dots .

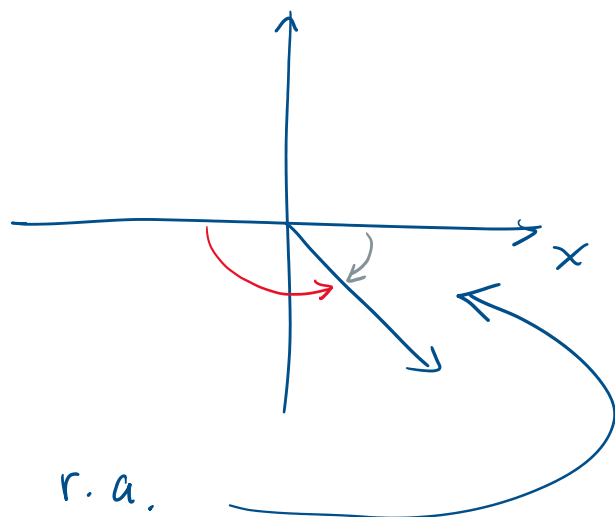
Remember: Angles go in counter-clockwise. ←

eg.



Only 1 angle, automatically a r. a..

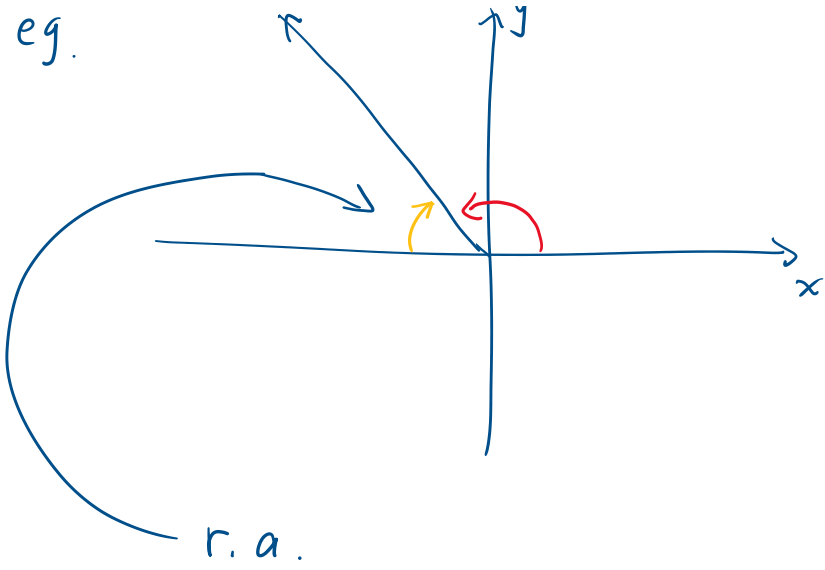
eg.



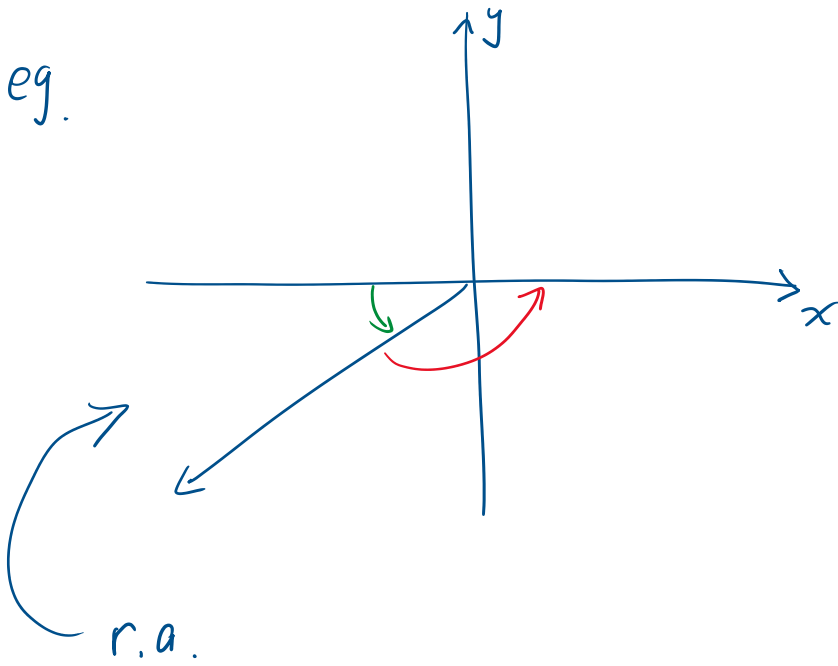
eg.



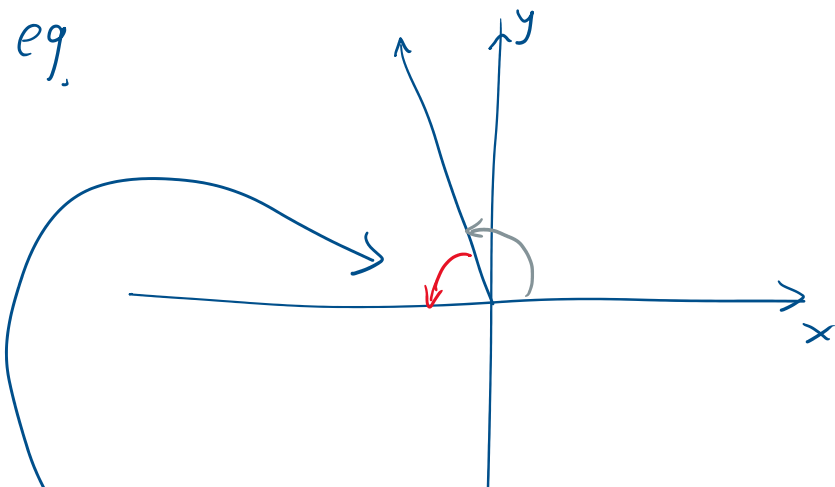
eg.



eg.

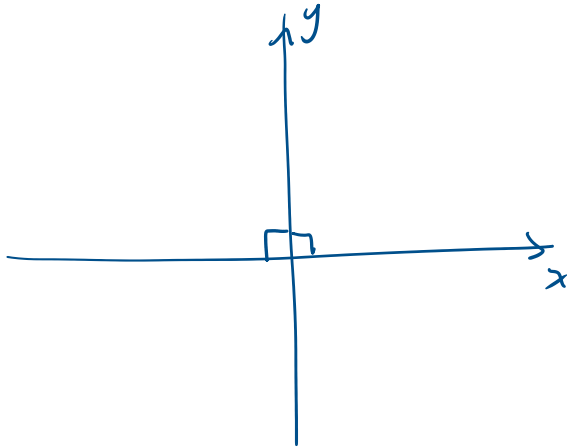


eg.

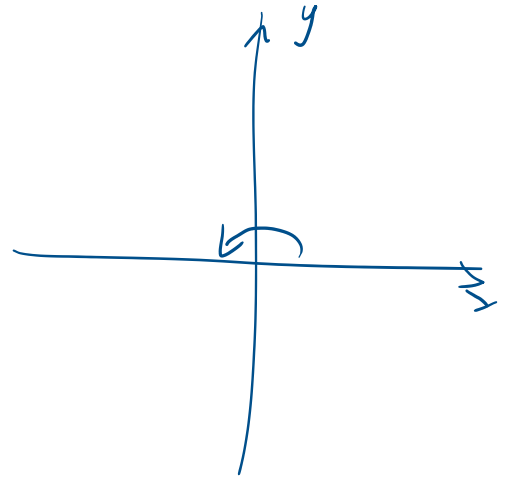




* eg.



r.a. : None



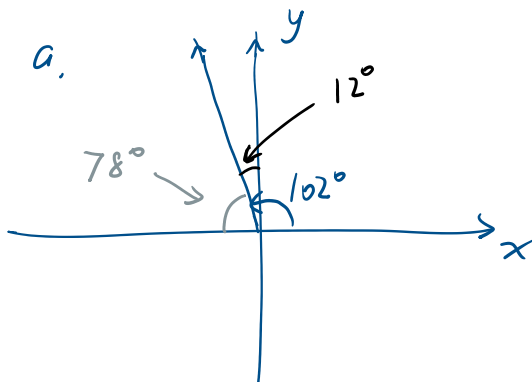
r.a. : None

eg. Find the reference angle:

a. 102°

b. 218°

Sol: a.



r.a. = $\boxed{78^\circ}$

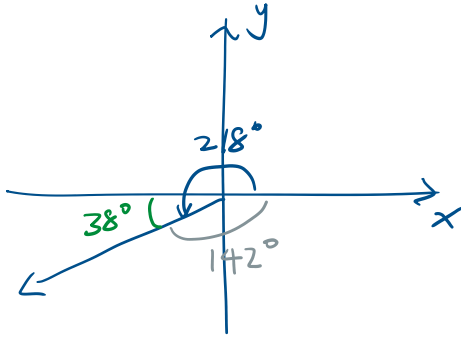
102° connects to 90°

$$102^\circ - 90^\circ$$

$$= 12^\circ$$

$$90^\circ - 12^\circ = 78^\circ$$

b.



218° connects to 180°

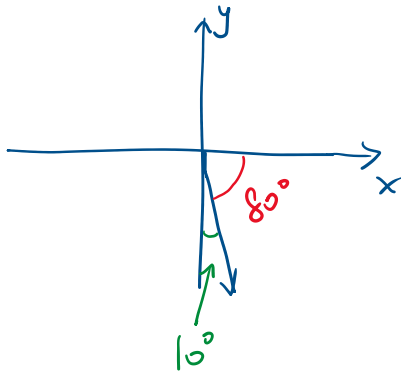
$$218^\circ - 180^\circ = 38^\circ \checkmark$$

$$180^\circ - 38^\circ = 142^\circ$$

$$\text{r.a.} = \boxed{38^\circ}$$

eg. Find a reference angle for an angle of 280° .

Sol:



280° connects to 270°

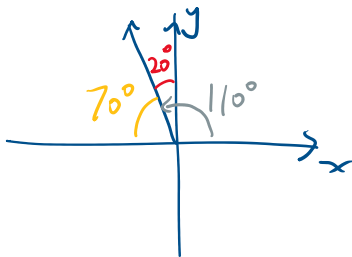
$$280^\circ - 270^\circ = 10$$

$$90^\circ - 10^\circ = 80$$

$$\text{r.a.} = \boxed{80^\circ}$$

eg. Find a reference angle for an angle of 470° .

Sol:



$$\text{r.a.} = 70^\circ$$

470° connects to 360° , because $>360^\circ$

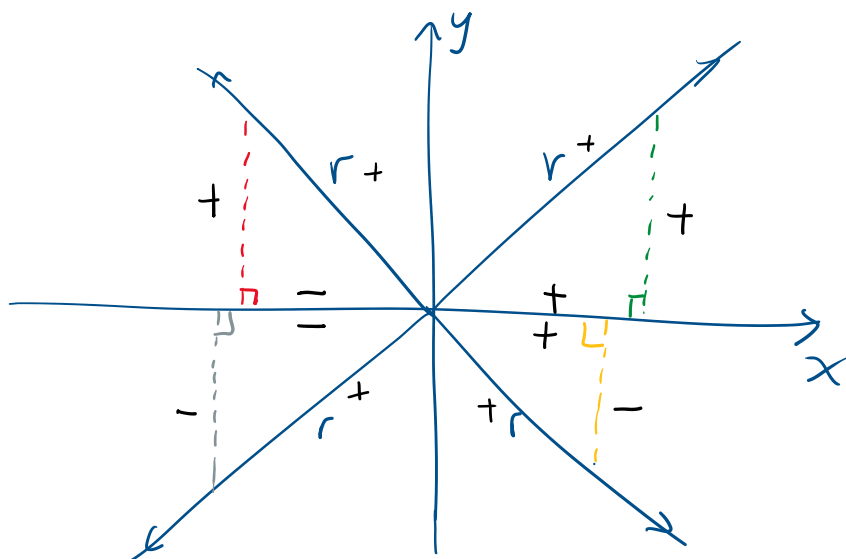
$$470^\circ - 360^\circ = 110^\circ$$

110° connects to 90°

$$110^\circ - 90^\circ = 20^\circ$$

$$90^\circ - 20^\circ = 70^\circ$$

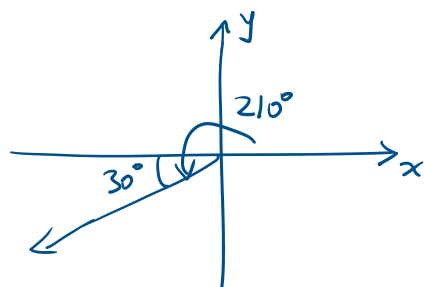
Now, because of the reference angle, we have the following:



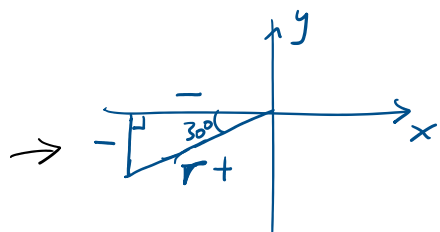
Remember the + or - from above

eg. Find the exact value (non-decimal) for $\sin 210^\circ$.

sol:



$$210^\circ - 180^\circ = 30$$

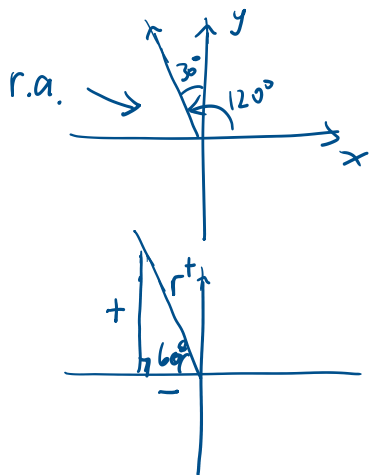


$$\sin 210^\circ = \sin 30^\circ \text{ in r.a.}$$

$$= \boxed{-\frac{1}{2}}$$

eg. Find the exact value of $\tan 120^\circ$

Sol:



$$120^\circ - 90^\circ = 30^\circ$$

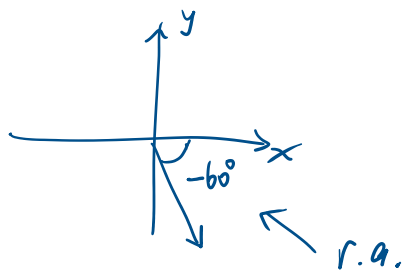
$$\text{r.a.} = 90^\circ - 30^\circ = 60^\circ$$

$$\tan 120^\circ = \tan 60^\circ \text{ in r.a.}$$

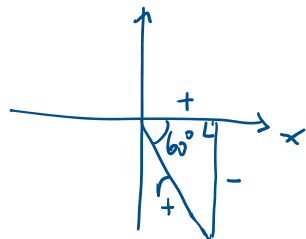
$$= \boxed{-\sqrt{3}}$$

eg. Find the exact value of $\cos 300^\circ$,

Sol:



$$300^\circ : 300^\circ - 360^\circ = -60^\circ$$



$$\cos 300^\circ = \cos 60^\circ \text{ in r.a.}$$

$$= \boxed{\frac{1}{2}}$$