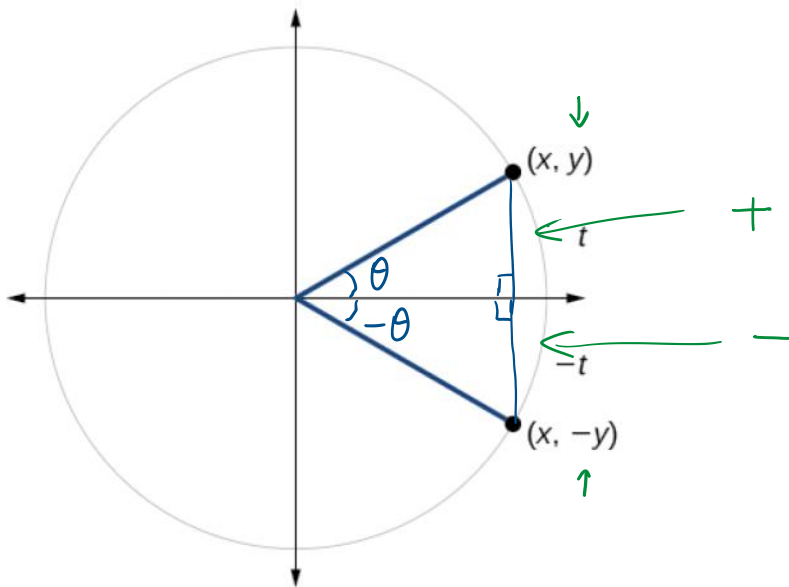


IX. Even-Odd Property



Algebra class:

Even fct: $f(-x) = f(x)$
 Odd fct: $f(-x) = -f(x)$

$$(-1)^{16} = 1$$

$$(-1)^{17} = -1$$

eg. Given a fct, $f(x) = x^3 - 8x$

$$f(-x) = (-x)^3 - 8 \cdot (-x)$$

$$= -x^3 - (-8x)$$

$$= -x^3 + 8x$$

$$= -1 \cdot (x^3 - 8x)$$

$$= -f(x)$$

$$\rightarrow = -f(x)$$

$f(x)$ is odd.

$$(-x)^3 \leftarrow$$

$$= (-1)^3 \cdot x^3$$

$$= -x^3$$

$$2x^2 + 4x - 6$$

$$= 2(x^2 + 2x - 3)$$

eg. Given a fct, $g(x) = 3x^4 - x^2 + 6$.

$$g(-x) = 3(-x)^4 - (-x)^2 + 6$$

$$= 3x^4 - x^2 + 6$$

$$= g(x)$$

$g(x)$ is even.

$$\begin{aligned} (-x)^4 &= (-1)^4 \cdot x^4 \\ &= 1 \cdot x^4 \\ &= x^4 \end{aligned}$$

eg. $h(x) = x^4 + x - 1$.

$$h(-x) = (-x)^4 + (-x) - 1$$

$$= x^4 - x - 1$$

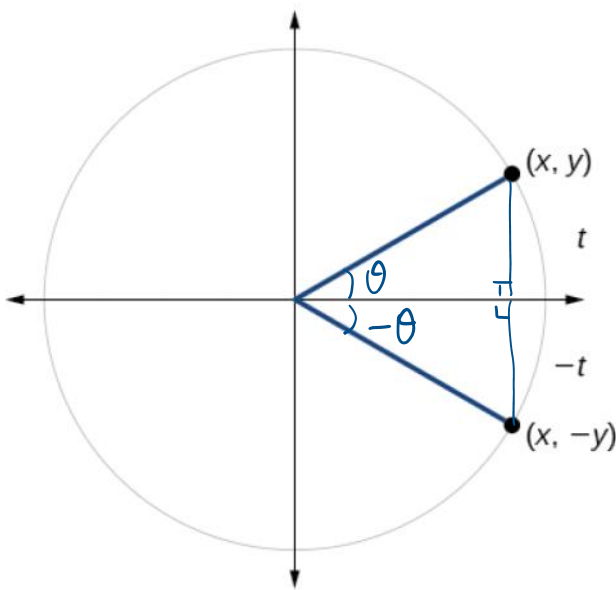
← not $x^4 + x - 1$

nor

$$-(x^4 + x - 1) = -x^4 - x + 1$$

← if even: $x^4 + x - 1$
if odd: $-(x^4 + x - 1)$
 $= -x^4 - x + 1$

$h(x)$ is neither even or odd.



← $\cos\theta$, $\sin\theta$, and $\tan\theta$ apply as a fct, respectively

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{odd}}{\text{even}} = \frac{-}{+} = -$$

∴ odd



odd	even	↙ odd
$\sin(-\theta) = -\sin\theta$	$\cos(-\theta) = \cos\theta$	$\tan(-\theta) = -\tan\theta$

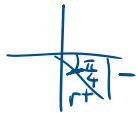
Because of above,

$\csc(-\theta) = -\csc\theta$	$\sec(-\theta) = \sec\theta$	$\cot(-\theta) = -\cot\theta$
↑	↑	↑
$\frac{1}{\sin(-\theta)}$	$\frac{1}{\cos(-\theta)}$	$\frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos\theta}{-\sin\theta} = -\frac{\cos\theta}{\sin\theta}$
		$= -\cot\theta$

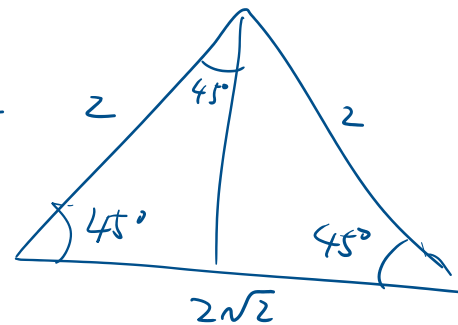
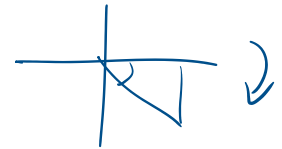
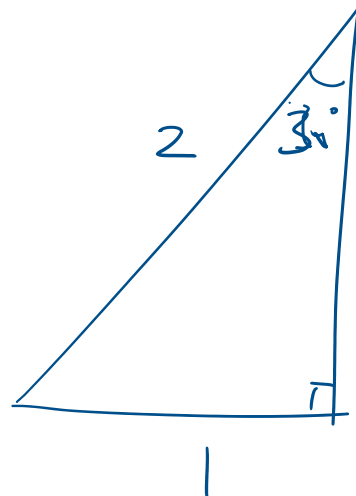
eg. Evaluation a. $\sin(-\frac{\pi}{4})$

b. $\cos(-\frac{\pi}{3})$

Sol: a. $= -\sin(\frac{\pi}{4})$
 $= -\frac{\sqrt{2}}{2}$



b. $= \cos(\frac{\pi}{3})$
 $= \frac{1}{2}$



eg. Evaluate $\sec(-\frac{\pi}{6})$.

Sol: $= \frac{1}{\cos(-\frac{\pi}{6})}$
 $= \frac{1}{\cos\frac{\pi}{6}}$
 $= \frac{1}{\frac{\sqrt{3}}{2}}$

$1 \cdot \frac{2}{\sqrt{3}}$

$$\begin{aligned} & \frac{2}{2} \\ &= \frac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} \\ &= \boxed{\frac{2\sqrt{3}}{3}} \end{aligned}$$

eg. Evaluate $\cot(-\frac{\pi}{4})$.

$$\begin{aligned} \text{sol: } &= \frac{1}{\tan(-\frac{\pi}{4})} \\ &= \frac{1}{-\tan \frac{\pi}{4}} \\ &= \frac{1}{-1} \\ &= \boxed{-1} \end{aligned}$$

eg. Evaluate $-3 \csc(-30^\circ) \sec(-60^\circ)$. ← $-3 \cdot \cancel{fct} \cdot \cancel{fct}$

$$\begin{aligned} \text{sol: } &= -3 \cdot \frac{1}{\sin(-30^\circ)} \cdot \frac{1}{\cos(-60^\circ)} && -4\sqrt{3} \\ &= -3 \cdot \frac{1}{-\sin(30^\circ)} \cdot \frac{1}{\cos(60^\circ)} && 12 \\ &= -3 \cdot \frac{1}{-\frac{1}{2}} \cdot \frac{1}{\frac{1}{2}} \\ &= -3 \cdot -2 \cdot 2 && \leftarrow 1 \cdot -\frac{2}{1} \\ &= 6 \cdot 2 && \leftarrow 1 \cdot 2 \\ &= \boxed{12} \end{aligned}$$