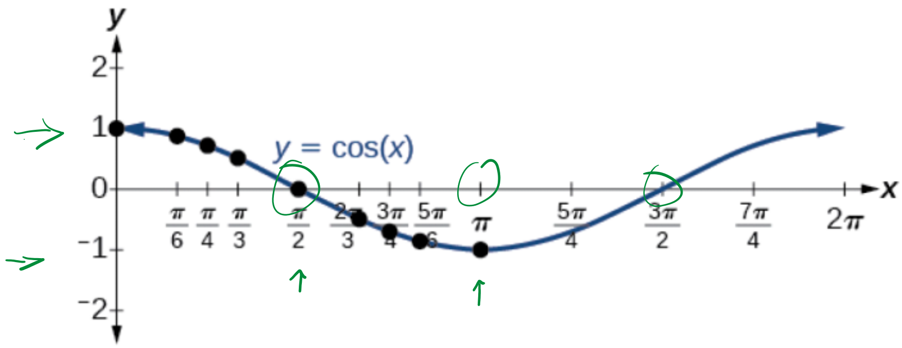
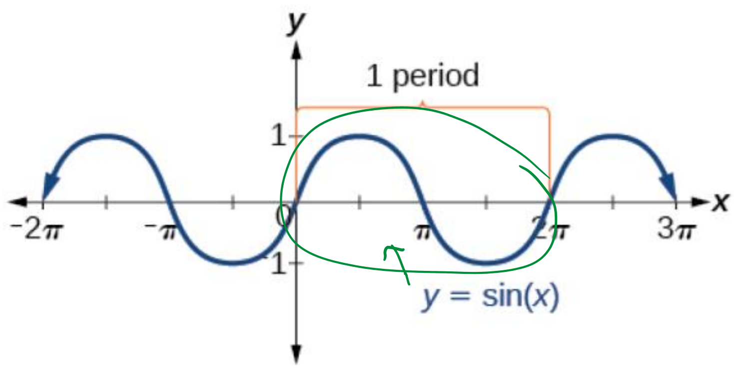
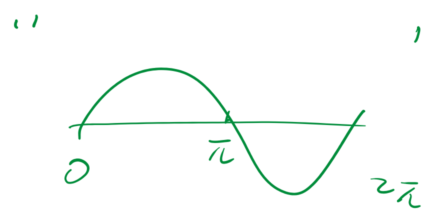
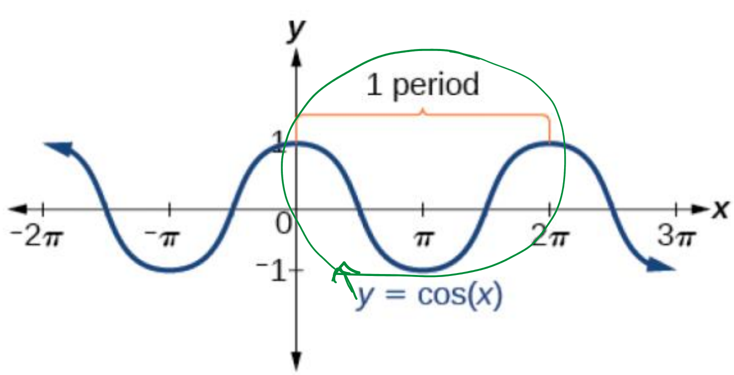
II. Cos x ← cosine

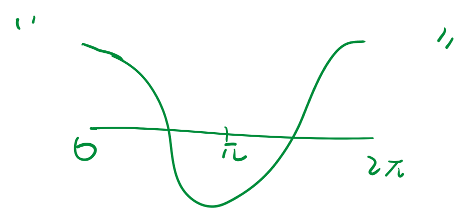
Cos x is similar to sin x, but the graph is somewhat different in terms of y-values and its periodic endpoints.

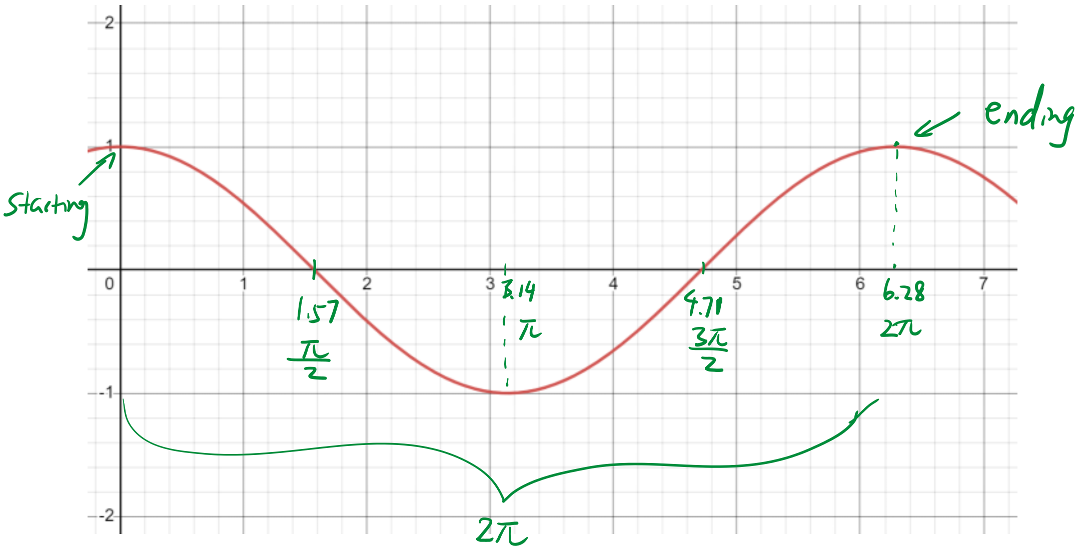


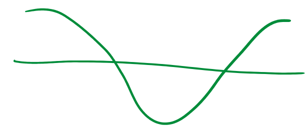








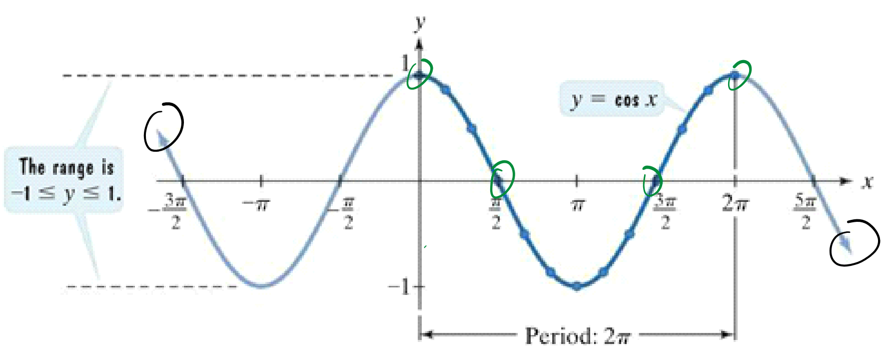




Similarly:

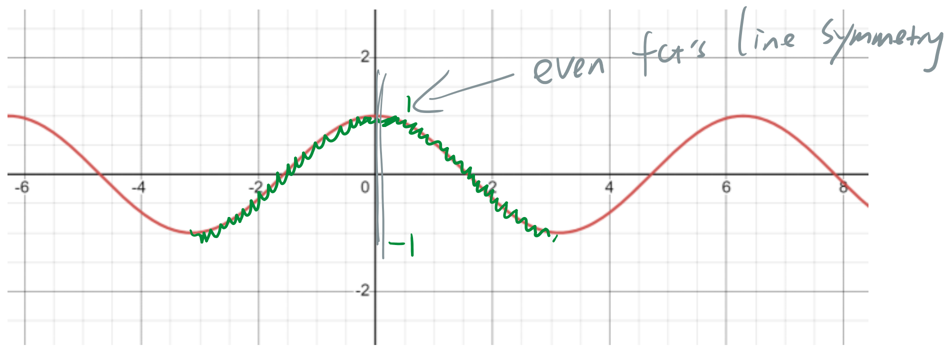
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\cdots$	π	$\cdots$	$\frac{3\pi}{2}$	$\cdots$	2π
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\cdots$	-1	$\cdots$	0	$\cdots$	1

← remember this, well!



The graph of $y = \cos x$ allows us to visualize some of the properties of the cosine function.

- ✓ The domain is $(-\infty, \infty)$, the set of all real numbers. The graph extends indefinitely to the left and to the right with no gaps or holes.
- ✓ The range is $[-1, 1]$, the set of all real numbers between -1 and 1, inclusive. The graph never rises above 1 or falls below -1.
- ✓ The period is 2π. The graph's pattern repeats in every interval of length 2π.
- ✓ The function is an even function: $\cos(-x) = \cos x$. This can be seen by observing that the graph is symmetric with respect to the y-axis.



eg. $\cos(-5x)$, then $\cos(-5x) = \cos(5x)$. ← to graph, it's important to turn $\cos(-5x)$ into $\cos(5x)$

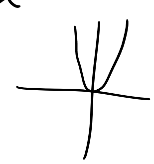
eg. If to graph $\cos(-2x)$, then we have $\cos(-2x) = \cos(2x)$, since $\cos(2x)$ is "easier" for graphing.

III. Transformation ← "shifts"

vertical shift

$$y = f(x) + c \quad -c,$$

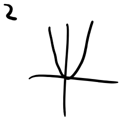
eg. $y = x^2 + 5$ ← c ← $y = x^2$ $+5$ up 5



horizontal shift

$$y = f(x + c) \quad -c$$

eg. $y = (x+3)^2$ ← $y = x^2$ $+3$ back 3



← "$x + 3 = 0$"
$-3 \quad -3$
$x = -3$ is "origin".

vertical stretch

$y = cf(x)$ for $c > 1$, "taller",

$c < 1$, "wider",

$\leftarrow$ from ... to ... ($y = x^2$ to $y = 2x^2$)

$\leftarrow$ from ... to ... ($y = \frac{1}{2}x^2$)

horizontal stretch

$y = f(cx)$, for $c > 1$, "taller",

$0 < c < 1$, "wider",

$c < 0$, algebra: odd, even, and work

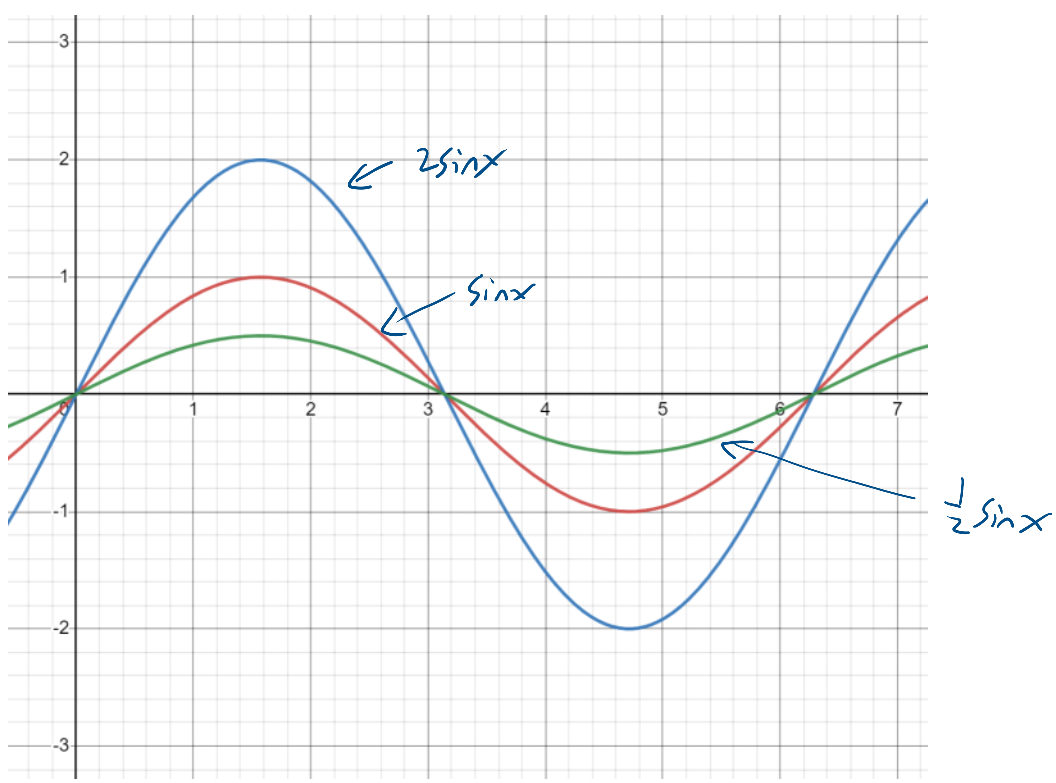
$\leftarrow$ $y = x^2$ and $y = (2x)^2 = 4x^2$

$\leftarrow$ $y = x^2$ and $y = (\frac{1}{2}x)^2 = \frac{1}{4}x^2$

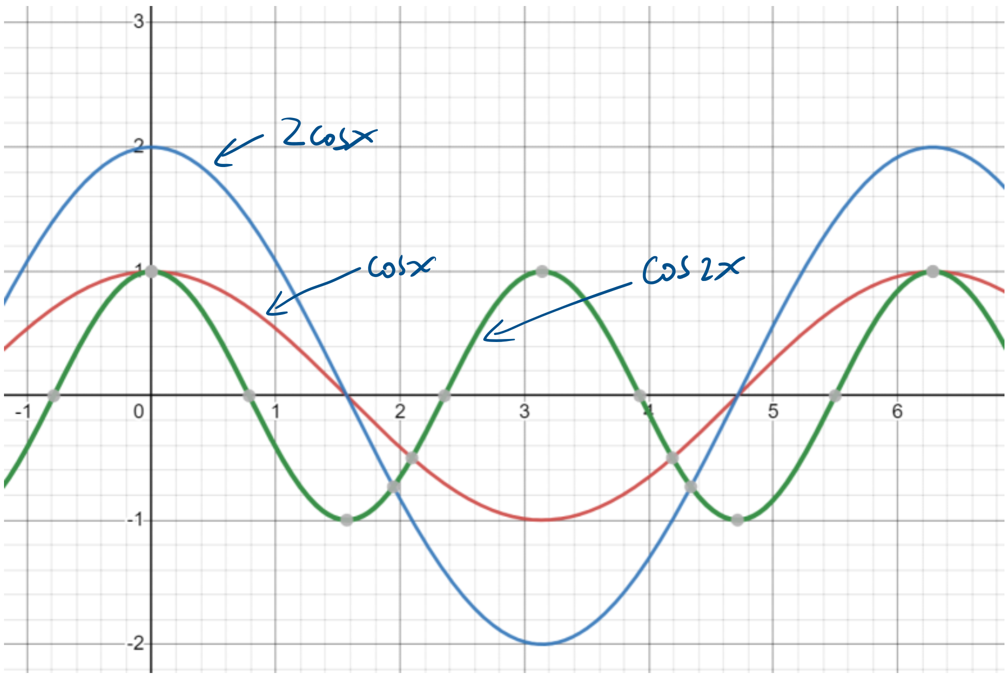
$\leftarrow$ $y = x^2$, $y = (-2x+5)^2$

algebra

eg.



eg.



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