

III. Cont.

eg. Graph $y = -2\sin 3x$ for one period.

Sol: Amplitude = $|-2| = 2$, period = $\frac{2\pi}{3}$

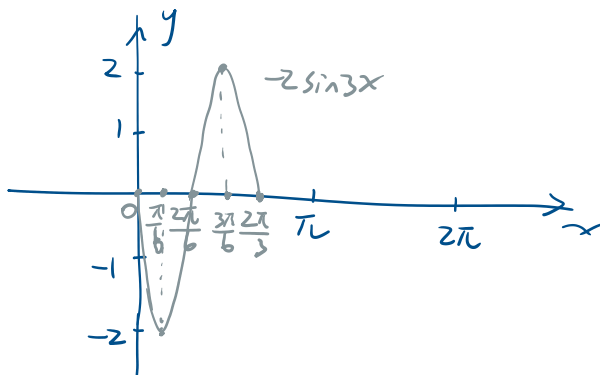
← ? * # = 2π

x	x_0	x_1	x_2	x_3	x_4
$3x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin 3x$	0	1	0	-1	0
$-2\sin 3x$	0	-2	0	2	0

← x's

← y's

← $\frac{2\pi}{3} = \frac{1}{3} \cdot 2\pi = \frac{2\pi}{3}$



Phase Shift 2

← Set $(\dots) = 0$

For the function of the type $y = \sin(x \pm d)$, where d is a number, we have $0 \leq x \pm d \leq 2\pi$

$$0 \pm d \leq x \leq 2\pi \pm d$$

Starting x ending x

Starting x

ending x

Step: 1. Find new period ' $\dots \leq x \leq \dots$ '

2. Repeat

x	\dots	\dots	\dots	x_4
\vdots				\vdots

3. Graph.

eg. Graph $y = \sin(x - \frac{\pi}{3})$ for one period.

Sol: Amplitude = $|1| = 1$, period:

$$0 \leq x - \frac{\pi}{3} \leq 2\pi$$

$$+\frac{\pi}{3} \quad \quad \quad +\frac{\pi}{3} \quad \quad \quad +\frac{\pi}{3}$$

$$\frac{\pi}{3} \leq x \leq \frac{7\pi}{3}$$

↑ starting ↑ ending

$$2\pi + \frac{\pi}{3}$$

$$= \frac{6\pi}{3} + \frac{\pi}{3}$$

$$= \frac{7\pi}{3}$$

	x_0	x_1	x_2	x_3	x_4
	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{8\pi}{6}$	$\frac{11\pi}{6}$	$\frac{7\pi}{3}$
$x - \frac{\pi}{3}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin(x - \frac{\pi}{3})$	0	1	0	-1	0

← x's

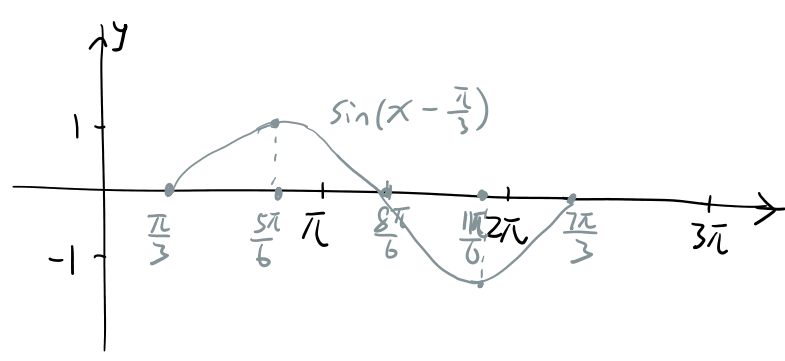
← y's

$$\frac{\frac{7\pi}{3} - \frac{\pi}{3}}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$

~~$$\frac{\pi}{3} + \frac{\pi}{2} = \frac{2\pi}{6} + \frac{3\pi}{6} = \frac{5\pi}{6}$$~~

~~$$\frac{5\pi}{6} + \frac{\pi}{2} = \frac{5\pi}{6} + \frac{3\pi}{6} = \frac{8\pi}{6}$$~~

~~$$\frac{8\pi}{6} + \frac{\pi}{2} = \frac{8\pi}{6} + \frac{3\pi}{6} = \frac{11\pi}{6}$$~~



$\sin(x - \frac{\pi}{3})$

$$x - \frac{\pi}{3} = 0$$

$$x = \frac{\pi}{3} \leftarrow \text{new origin}$$

← last vertical shift

eg. Graph the function $y = 3 \cos(x + \frac{\pi}{4}) - 1$ for one period.

Sol: Amplitude = $|3| = 3$, period: $0 \leq x + \frac{\pi}{4} \leq 2\pi$

$$-\frac{\pi}{4} \qquad -\frac{\pi}{4} \qquad -\frac{\pi}{4}$$

$$-\frac{\pi}{4} \leq x \leq \frac{7\pi}{4}$$



x	x_0	x_1	x_2	x_3	x_4
	$-\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$
$x + \frac{\pi}{4}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos(x + \frac{\pi}{4})$	1	0	-1	0	1
$3 \cos(x + \frac{\pi}{4})$	3	0	-3	0	3
$3 \cos(x + \frac{\pi}{4}) - 1$	2	-1	-4	-1	2

← x's

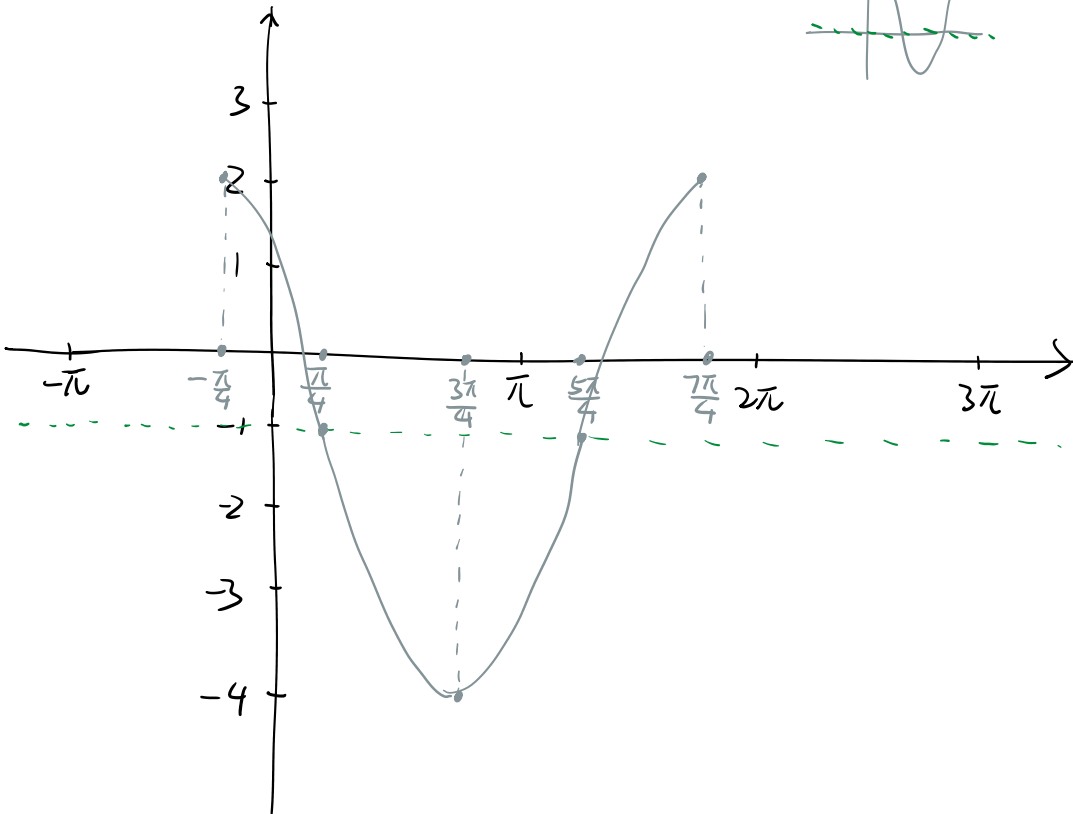
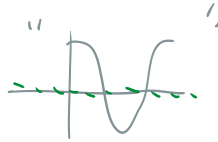
$$\frac{2\pi - (-\frac{\pi}{4})}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$



← y's

~~$$-\frac{\pi}{4} + \frac{2\pi}{4} = \frac{\pi}{4}$$~~
~~$$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$$~~

~~$$\frac{\pi}{4} + \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$~~
~~$$\frac{3\pi}{4} + \frac{\pi}{4} = \frac{4\pi}{4} = \pi$$~~



← "x-axis"