

III. Cont.



eg. Graph  $y = 2\sin(\frac{1}{2}x - \frac{\pi}{4}) - 1$ .

← inside  $\frac{1}{2}x - \frac{\pi}{4}$

sol: Amplitude =  $|2| = 2$ , period:  $0 \leq \frac{1}{2}x - \frac{\pi}{4} \leq 2\pi$

$\frac{\pi}{4} \leq \frac{1}{2}x \leq \frac{9\pi}{4}$

$\frac{\pi}{2} \leq x \leq \frac{9\pi}{2}$

$\frac{\pi}{4} \cdot 2 = \frac{\pi}{2}$   
 $\frac{9\pi}{4} \cdot 2 = \frac{9\pi}{2}$

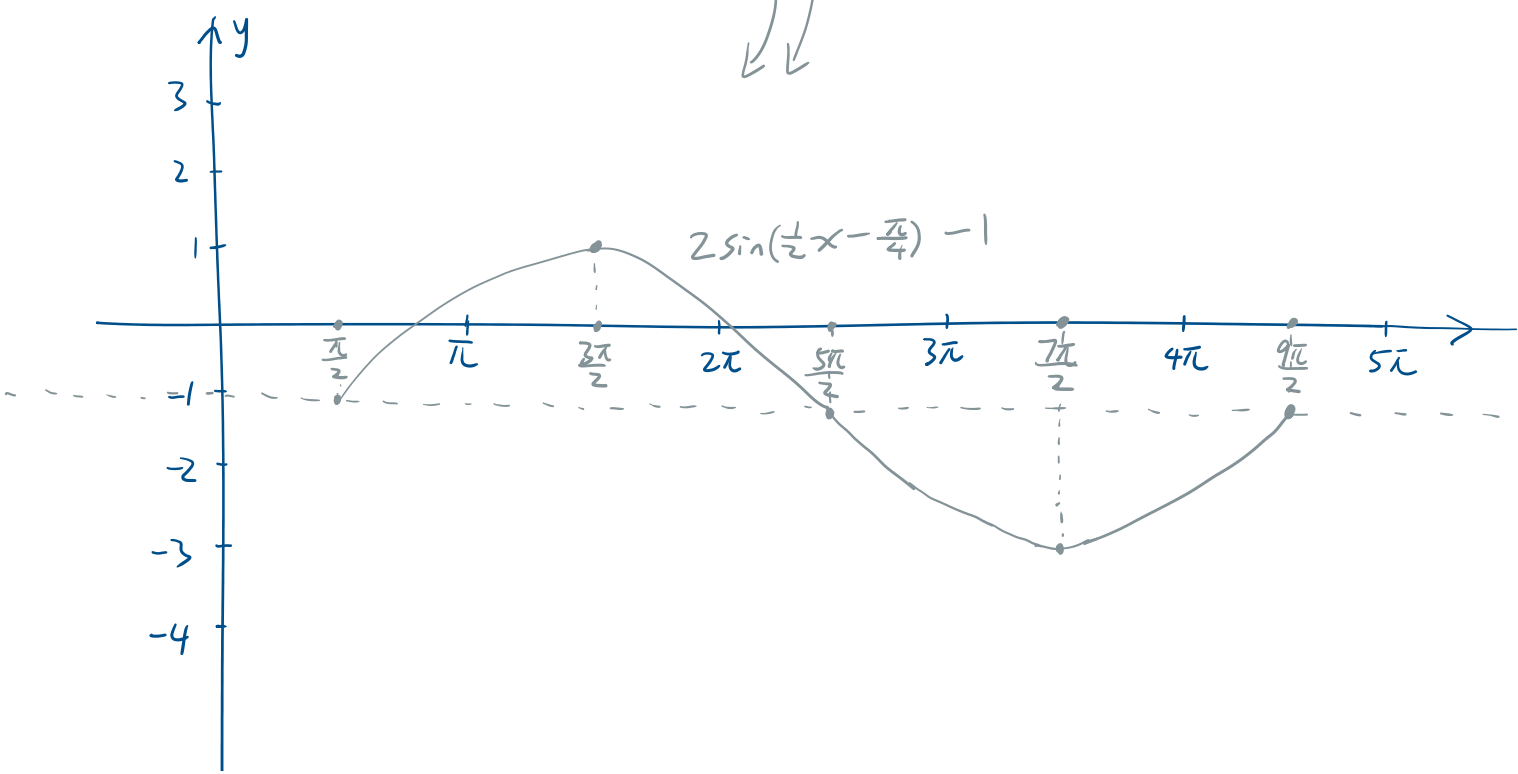


$x$	$x_0$	$x_1$	$x_2$	$x_3$	$x_4$
$x$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{5\pi}{2}$	$\frac{7\pi}{2}$	$\frac{9\pi}{2}$
$\frac{1}{2}x$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{9\pi}{4}$
$\frac{1}{2}x - \frac{\pi}{4}$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
$\sin(\frac{1}{2}x - \frac{\pi}{4})$	0	1	0	-1	0
$2\sin(\frac{1}{2}x - \frac{\pi}{4})$	0	2	0	-2	0
$2\sin(\frac{1}{2}x - \frac{\pi}{4}) - 1$	-1	1	-1	-3	-1

$\frac{9\pi}{2} - \frac{\pi}{2} = \frac{8\pi}{2} = \frac{4\pi}{1} = \pi$

$\frac{\pi}{2} \quad \pi - \frac{\pi}{2} + \frac{2\pi}{2} - \frac{3\pi}{2}$

1                    1                    2                    1



## IV. Tangent and Cotangent

### i. Tan

The tangent fct is odd fct, that is

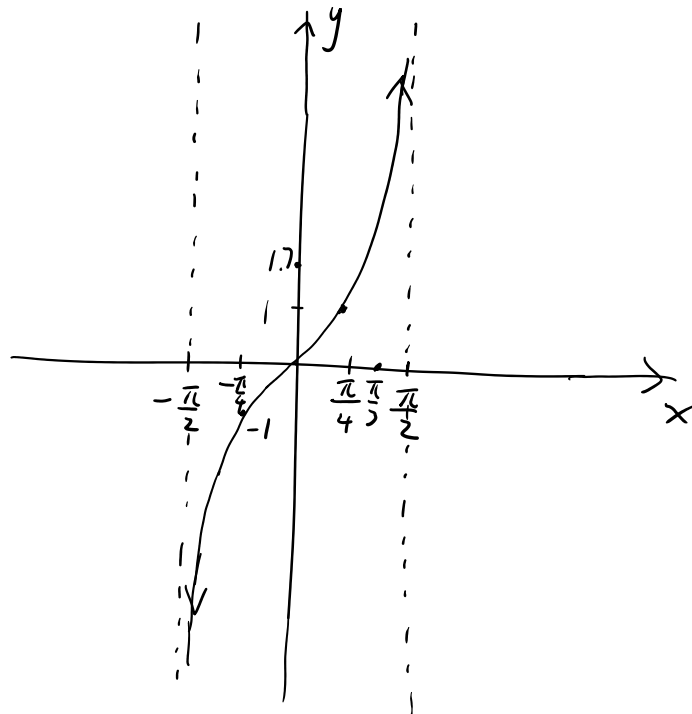
$$f(x) = \tan x$$

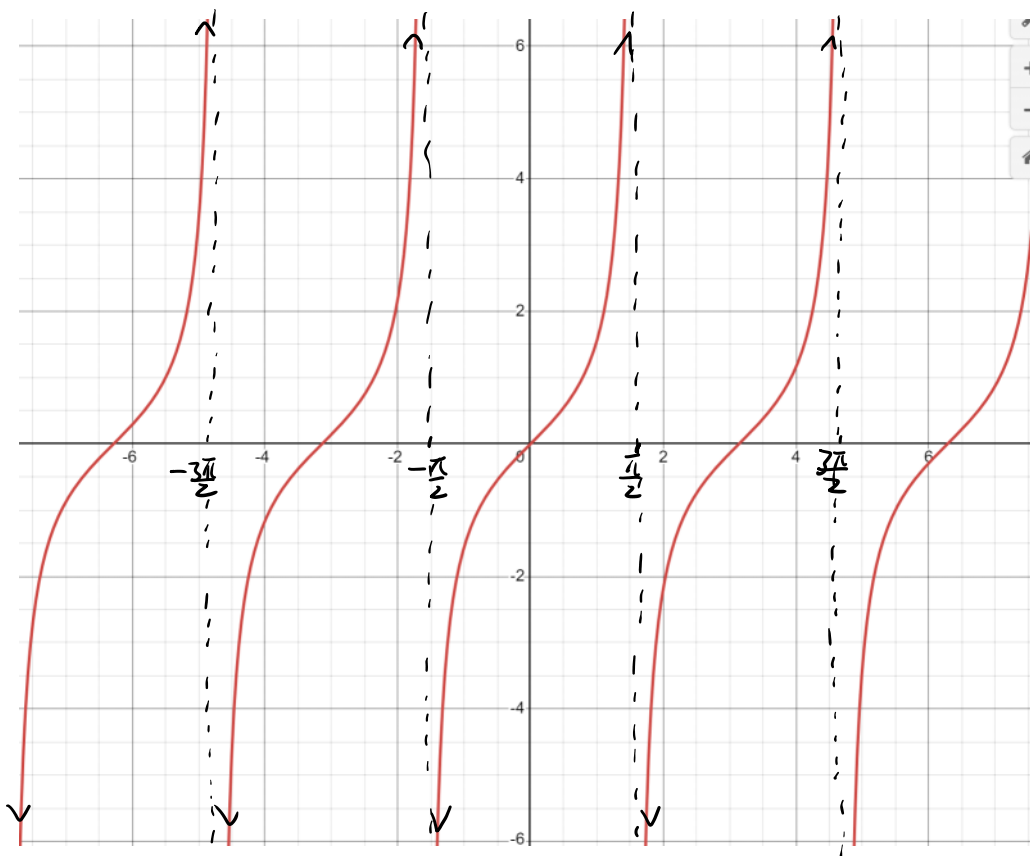
$$f(-x) = -\tan x$$

eg.  $\tan(-25\pi) = -\tan(25\pi)$

x	y
$-\frac{\pi}{2}$	$-\infty$
...	...
0	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \approx 0.6$
...	...
$\frac{\pi}{4}$	1
...	...
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.7$
...	...
$\frac{\pi}{2}$	$+\infty$

One period:





Domain:  $(-\infty, \infty) - \frac{\pi}{2}n$ , where  $n$  is an integer.

Range:  $(-\infty, \infty)$

### transformation

All of the previous properties apply. We focus on  $-\frac{\pi}{2}, -\frac{\pi}{4}, \dots, \frac{\pi}{2}$ .

eg. Graph  $y = \tan 2x - 1$ .

Sol:  $\downarrow$   
 $\tan 2x - 1$ , no scaler

period:  $-\frac{\pi}{2} \leq \frac{2x}{2} \leq \frac{\pi}{2}$

$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

tan's period  
 $\swarrow \quad \searrow$   
 $-\frac{\pi}{2} \text{ to } \frac{\pi}{2}$

x	$x_0$	$x_1$	$x_2$	$x_3$	$x_4$
	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$

$\frac{\frac{\pi}{4} - (-\frac{\pi}{4})}{4} = \frac{\frac{2\pi}{4}}{4} = \frac{\pi}{8}$

$x$	$-\frac{\pi}{4}$	$-\frac{\pi}{8}$	$0$	$\frac{\pi}{8}$	$\frac{\pi}{4}$
$2x$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	$0$	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\tan 2x$	$-\infty$	$-1$	$0$	$1$	$\infty$
$\tan 2x - 1$	$-\infty$	$-2$	$-1$	$0$	$\infty$

$$\frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{8}$$

~~$$-\frac{\pi}{4} + \frac{\pi}{8} = -\frac{\pi}{8} + \frac{\pi}{8}$$

$$-\frac{\pi}{8} + \frac{\pi}{8} = 0$$~~

