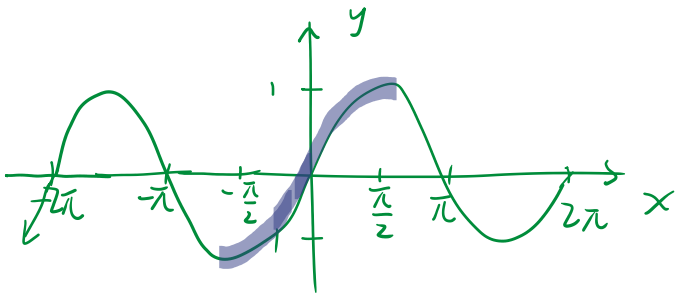
V. Inverse Fct

i. $\sin^{-1}x$ ← another name $\arcsin x$

Given the $\sin x$, we have the domain and range:



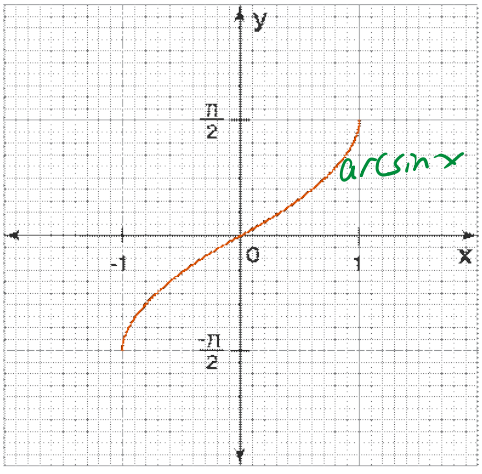
It is $x = \sin y$

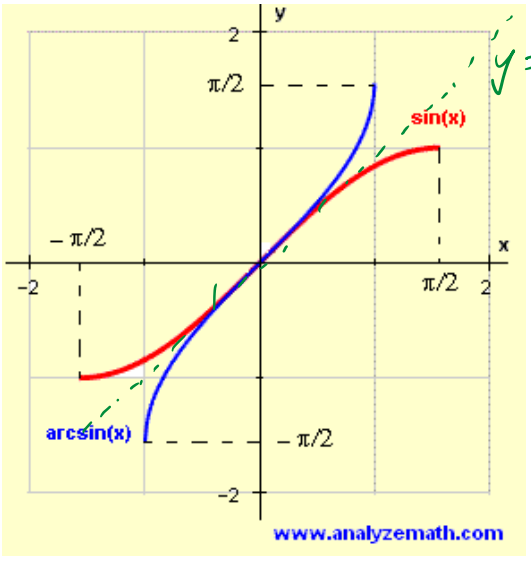
$\Rightarrow \sin^{-1}x = y$ ← domain and range switched.

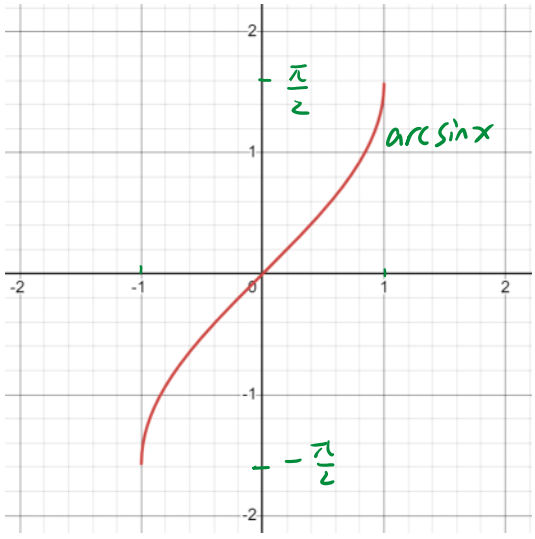
That is, the inverse of $\sin^{-1}x$ ($\arcsin x$) has domain: $-1 \le x \le 1$ and range of $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Arcsin Graph

cuemath
THE MATH EXPERT







eg. Find the exact value of $\sin^{-1}\frac{\sqrt{2}}{2}$.

Sol: $\sin^{-1}\frac{\sqrt{2}}{2}$ means $\sin ? = \frac{\sqrt{2}}{2}$ and it's within $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\Rightarrow \sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \boxed{45^\circ}$ or $\boxed{\frac{\pi}{4}}$

or:

$\sin^{-1}\frac{\sqrt{2}}{2} = x$

$\sin\sin^{-1}\frac{\sqrt{2}}{2} = \sin x$

$\frac{\sqrt{2}}{2} = \sin x$

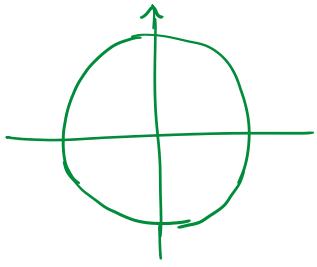
$\boxed{\frac{\pi}{4}}$ or $\boxed{45^\circ} = x$

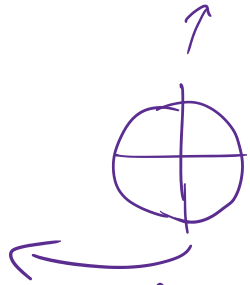
eg. Find the values of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ from $0 \le \theta \le 2\pi$.

Sol: $\sin ? = -\frac{\sqrt{3}}{2}$

$\sin ? = \frac{\sqrt{3}}{2}$

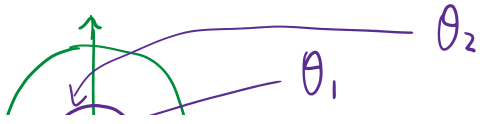
$\frac{\pi}{3}$

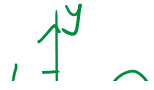




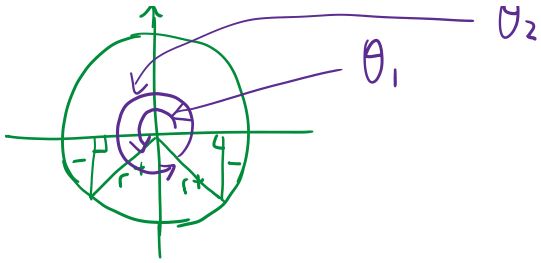
Two θ's produce $-\frac{\sqrt{3}}{2}$

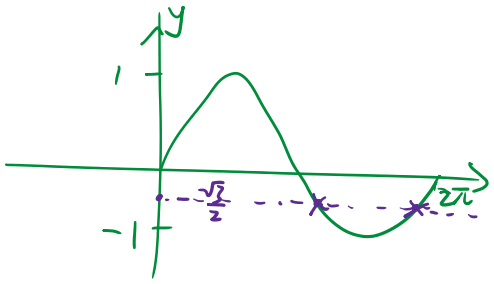
$-\frac{\sqrt{3}}{2} \Rightarrow$





$-\frac{\sqrt{3}}{2} \Rightarrow$





$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = \pi + \frac{\pi}{3}, \quad 2\pi - \frac{\pi}{3}$

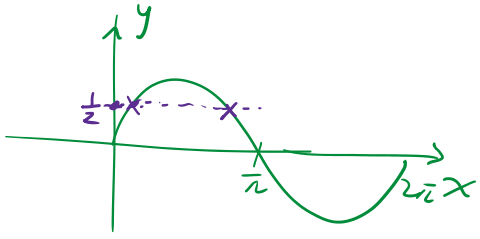
$\theta_1 = \boxed{\frac{4\pi}{3}}, \quad \theta_2 = \boxed{\frac{5\pi}{3}}$

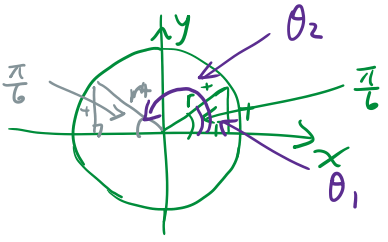
eg. Find the values of $\sin^{-1}\left(\frac{1}{2}\right)$ from $0 \le \theta \le 2\pi$.

Sol:

$\sin ? = \frac{1}{2}$

$\sin\theta = \frac{1}{2} \longleftarrow 0 \le \theta \le 2\pi$





$\sin ? = \frac{1}{2}$

$\frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \quad \pi - \frac{\pi}{6}$

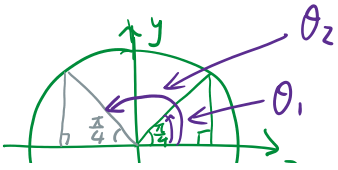
$\theta_1 = \boxed{\frac{\pi}{6}}, \quad \theta_2 = \boxed{\frac{5\pi}{6}}$

eg. Find the values of $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ for any periods.

Sol:

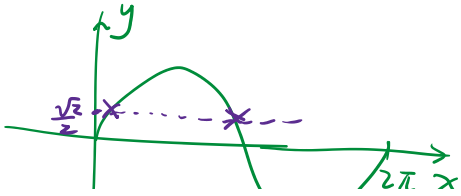
$\sin ? = \frac{\sqrt{2}}{2}$

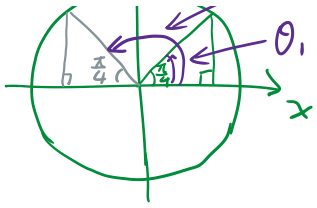
$\sin\theta = \frac{\sqrt{2}}{2}$

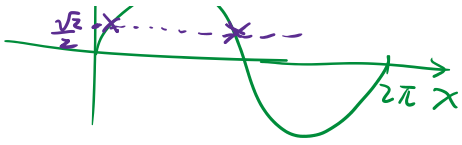


$\sin ? = \frac{\sqrt{2}}{2}$

$\frac{\pi}{4}$







$\theta = \frac{\pi}{4}, \pi - \frac{\pi}{4}$

$\theta_1 = \frac{\pi}{4}, \quad \theta_2 = \frac{3\pi}{4}$

$2\pi n \cdot \frac{4}{4}$

Any periods: $\theta_1 = \boxed{\frac{\pi}{4} + 2\pi n}$, $\theta_2 = \boxed{\frac{3\pi}{4} + 2\pi n}$ for any integers n

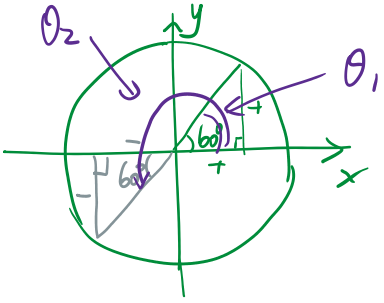
or: $= \frac{\pi}{4} + \frac{8\pi n}{4}$ $= \frac{3\pi}{4} + \frac{8\pi n}{4}$

$= \boxed{\frac{8\pi n + \pi}{4}}$ $= \boxed{\frac{8\pi n + 3\pi}{4}}$

eg. Find the values of $\tan^{-1}\sqrt{3}$ for any periods.

Sol: $\tan ? = \sqrt{3}$

$\tan\theta = \sqrt{3}$



$\tan ? = \sqrt{3}$

$60°$

$\theta_1 = 60°, \quad \theta_2 = 240°$

Period: $\boxed{180° = \pi}$

$\theta_1 = \boxed{60° + 180°n}$, $\theta_2 = \boxed{240° + 180°n}$

$\left(\theta_1 = \boxed{\frac{\pi}{3} + \pi n}, \quad \theta_2 = \boxed{\frac{4\pi}{3} + \pi n}\right)$