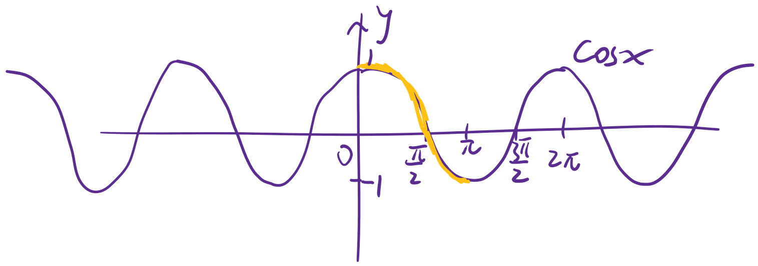
V. ii. $\cos^{-1}x$

The inverse of $\cos x$ is from:



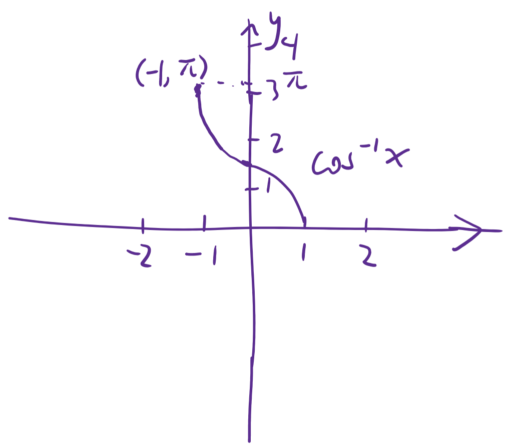
domain: $0 \le x \le \pi$

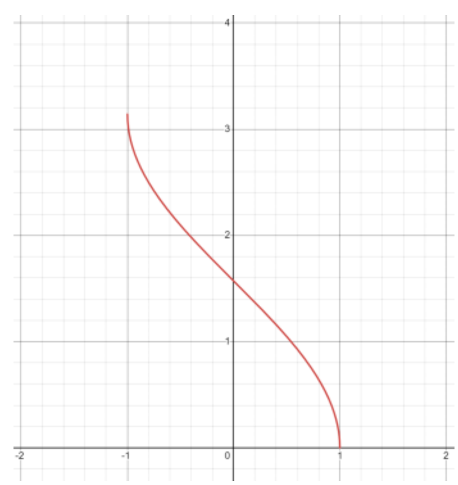
range: $-1 \le y \le 1$

Thus, $y = \cos^{-1}x$ has the

domain: $-1 \le x \le 1$

range: $0 \le y \le \pi$



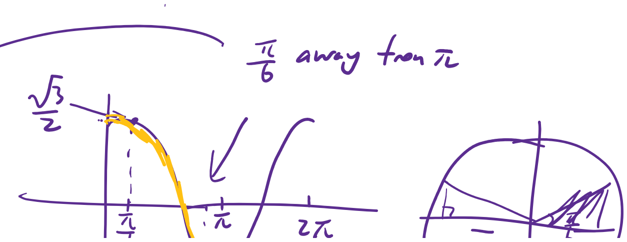


eg. Find the exact value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

Sol: $\cos ? = -\frac{\sqrt{3}}{2}$

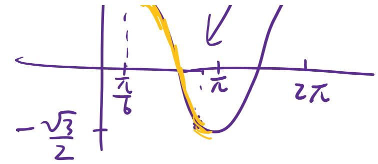
$\cos\theta = -\frac{\sqrt{3}}{2}$

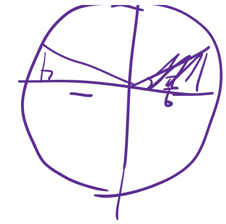
$A = \boxed{\frac{5\pi}{6}}$



$\cos\theta = - \frac{\quad}{2}$

$\theta = \boxed{\frac{5\pi}{6}}$





Note: There is only 1 inverse.

$\cos ? = \frac{\sqrt{3}}{2}$

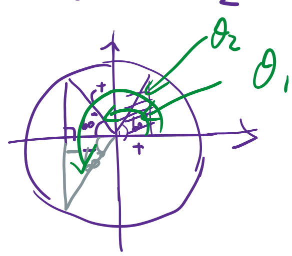
$30°, \ \frac{\pi}{6}$

eg. Find the exact value of $\cos^{-1}(-\frac{1}{2})$ from $0 \le \theta \le 2\pi$.

Sol:

$\cos ? = -\frac{1}{2}$

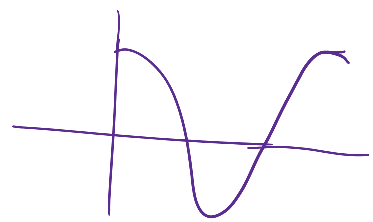
$\cos\theta = -\frac{1}{2}$



$\cos ? = \frac{1}{2}$

$60°, \ \frac{\pi}{3}$

$= -\frac{1}{2}$



$\theta_1 = \pi - \frac{\pi}{3}, \quad \theta_2 = \pi + \frac{\pi}{3}$

$\theta_1 = \boxed{\frac{2\pi}{3}}, \quad \theta_2 = \boxed{\frac{4\pi}{3}}$

eg. Find the exact value of $\cos^{-1}\frac{2\pi}{3}$. ← tricky

Sol:

$\cos ? = \frac{2\pi}{3}$

$\cos\theta = \frac{2\pi}{3}$

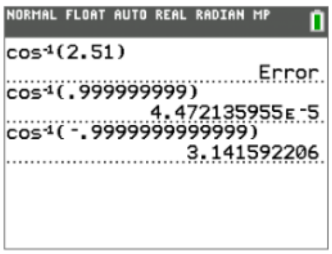
$\frac{2\pi}{3} > 1$, that is undefined.

$\boxed{\text{Undefined}}$

eg. Evaluate $\cos^{-1} 2.51$.

d. Evaluate $\cos^{-1} 2.51$.

Sol: $\boxed{\text{Undefined}}$


NORMAL FLOAT AUTO REAL RADIAN MP

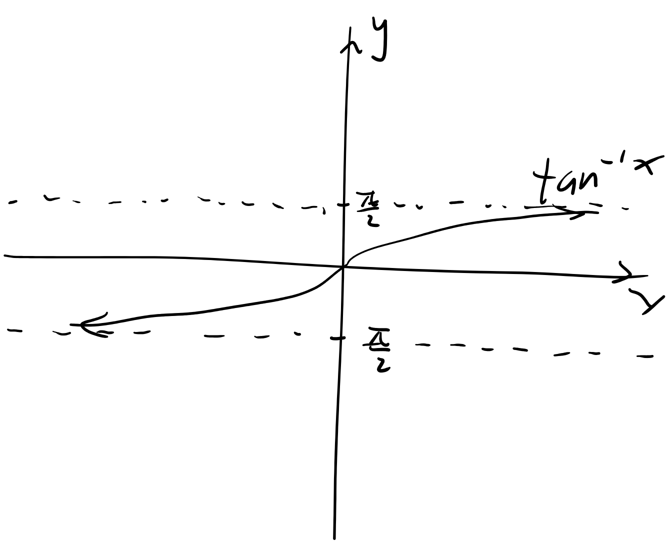
cos⁻¹(2.51)
Error
cos⁻¹(.999999999)
4.472135955E-5
cos⁻¹(⁻.9999999999999)
3.141592206

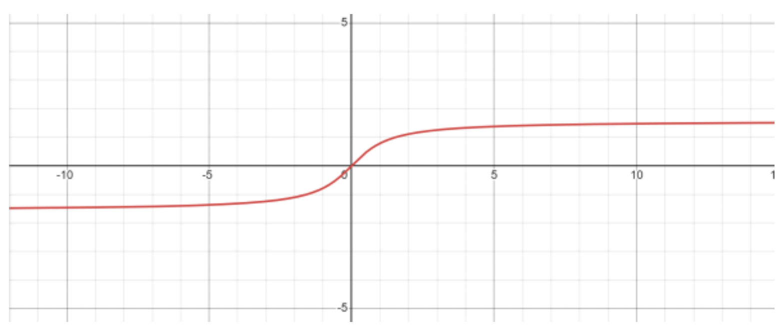
iii. $\tan^{-1}x$

Similarly, the inverse has

domain: $-\infty < x < \infty$,

range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$





eg. Find the exact value of $\tan^{-1}\sqrt{3}$.

Sol:

$\tan ? = \sqrt{3}$

$\tan\theta = \sqrt{3}$

$\boxed{\theta = 60^\circ}$

$\tan\theta = \sqrt{3}$

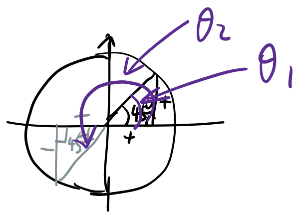
$\theta = \boxed{60^\circ}$

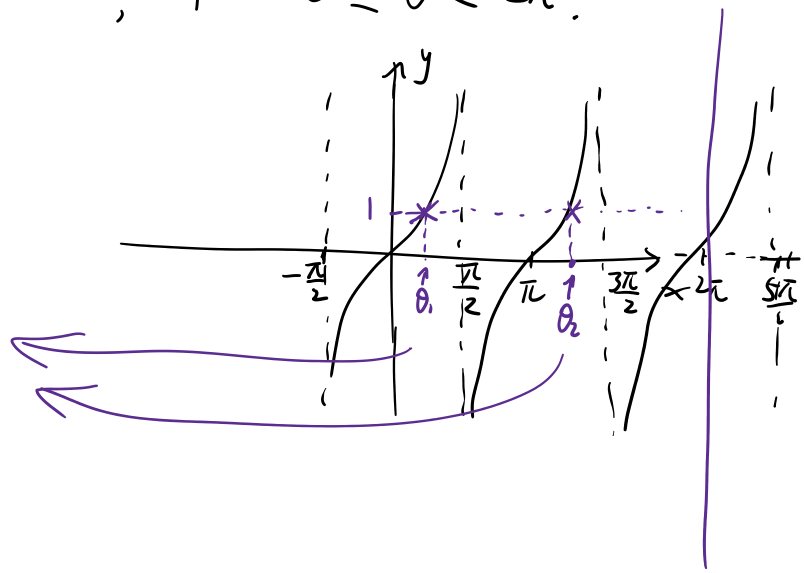
eg. Find the exact value of $\tan^{-1} 1$, for $0 \le \theta \le 2\pi$.

Sol:

$\tan ? = 1$

$\tan\theta = 1$





$\theta_1 = \frac{\pi}{4}, \quad \theta_2 = \pi + \frac{\pi}{4}$

$\theta_1 = \boxed{\frac{\pi}{4}}, \quad \theta_2 = \boxed{\frac{5\pi}{4}}$