14. Solve $\tan\theta = 2\sin\theta$ within $[0, 2\pi)$.

Sol: $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

then $\cancel{\cos\theta} \cdot \dfrac{\sin\theta}{\cancel{\cos\theta}} = 2\sin\theta \cdot \cos\theta$

2sinθcosθ

$\sin\theta = 2\sin\theta\cos\theta$

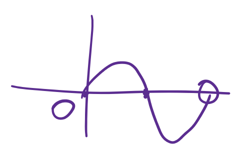
$\sin\theta = \sin 2\theta$

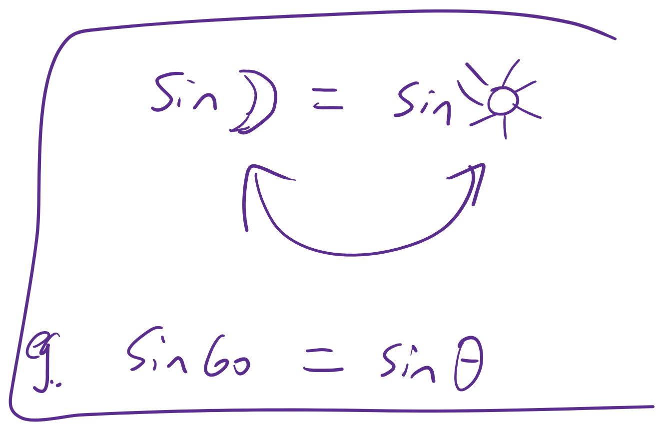
$\theta = 2\theta$

$-\theta \quad -\theta$

$0 = \theta$ ← only 1 of them

$\theta = \boxed{0^\circ, 180^\circ}$





15. $2\cos^2\theta + 1 = 3\cos\theta$

Sol: $-3\cos\theta \quad -3\cos\theta$

$2\cos^2\theta - 3\cos\theta + 1 = 0$

$(2\cos\theta - 1)(\cos\theta - 1) = 0$ ←

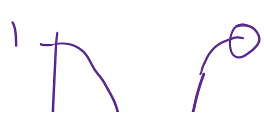
$\begin{array}{ccc} 2 & & -1 \\ & \times & \\ 1 & & -1 \end{array}$

$2\cos\theta - 1 = 0, \quad \cos\theta - 1 = 0$

$+1 \quad +1 \qquad +1 \quad +1$

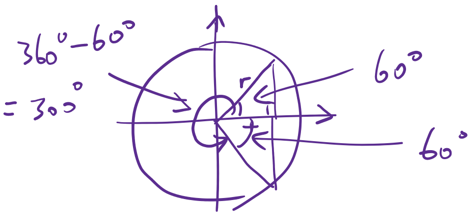
$\dfrac{\cancel{2}\cos\theta}{\cancel{2}} = \dfrac{1}{2} \qquad \cos\theta = 1$

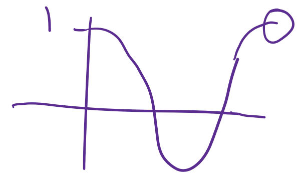
$\cos\theta = \frac{1}{2}$



← "easier"

$\cos\theta = \frac{1}{2}$





← "easier"

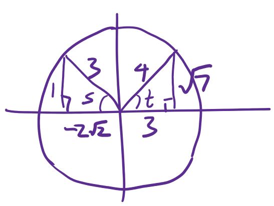
$\theta = 0^\circ$

$\theta = 60^\circ, 300^\circ$

$\theta = \boxed{0^\circ, 60^\circ, 300^\circ}$

Let $\sin s = \frac{1}{3}$, with s in quadrant II and let $\cos t = \frac{3}{4}$, with t in quadrant I. Find each of the following.

4. $\cos(s + t)$



$4^2 = 3^2 + y^2$

$16 = 9 + y^2$

$-9 \quad -9$

$7 = y^2$

$\pm\sqrt{7} = y$

$1^2 + x^2 = 3^2$

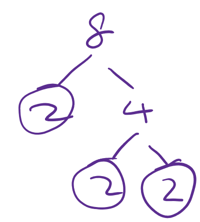
$1 + x^2 = 9$

$-1 \quad -1$

$x^2 = 8$

$\sqrt{x^2} = \sqrt{8}$

$x = \pm 2\sqrt{2}$



$8 = 2 \cdot 2 \cdot 2$

$= 2^3$

$\sqrt{8} = \sqrt{2^3} = \sqrt{2^2 \cdot 2}$

$= 2\sqrt{2}$

$\cos(s+t) = \cos s \cos t - \sin s \sin t$

$= \frac{-\cancel{2}^{1}\sqrt{2}}{_{1}\cancel{3}} \cdot \frac{\cancel{3}^{1}}{\cancel{4}_{2}} - \frac{1}{3} \cdot \frac{\sqrt{7}}{4}$

$= -\frac{\sqrt{2}}{2} - \frac{\sqrt{7}}{12}$

$= -\frac{6\sqrt{2}}{12} - \frac{\sqrt{7}}{12}$

$= \boxed{\frac{-6\sqrt{2} - \sqrt{7}}{12}}$