I. Trig Identities

Early Trig Identity:

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

odd/even: $\sin x$ and $\tan x$ are odd. $\cos x$ is even.

eg. $\sin(-2x) = -\sin(2x)$

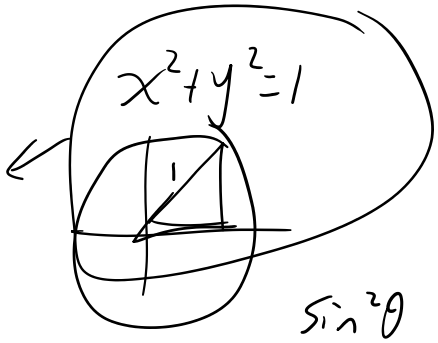
$\cos(-2x) = \cos(2x)$

$\tan(-2x) = -\tan(2x)$

$$\tan(-2x) = \frac{\sin(-2x)}{\cos(-2x)} = \frac{-\sin(2x)}{\cos(2x)} = -\frac{\sin(2x)}{\cos(2x)} = -\tan(2x)$$

Pythagorean Identity:

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$



$\sin^2\theta$ is $(\sin\theta)^2$

New:

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

← do whatever it takes to remember

Pf: $\tan^2\theta + 1 = \left(\frac{\sin\theta}{\cos\theta}\right)^2 + 1$

$= \frac{\sin^2\theta}{\cos^2\theta} + 1$

$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}$ ← LCD is $\cos^2\theta$; ← set up equivalent fraction

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$

$= \left(\frac{1}{\cos\theta}\right)^2$ ↘ $\frac{1^2}{\cos^2\theta}$

$= \sec^2\theta$

eg. Verify the identity $\sec\theta\cot\theta = \csc\theta$.

Sol: $\sec\theta = \frac{1}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$, expect $\csc\theta = \frac{1}{\sin\theta}$

then, $\frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta}$

$= \csc\theta$ ✓

eg. Verify $\sin\theta\tan\theta + \cos\theta = \sec\theta$.

RHS is $\sec\theta$ only

expect $\sec\theta = \frac{1}{\cos\theta}$

Sol: $\sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta$ ← good to try

$= \frac{\sin^2\theta}{\cos\theta} + \cos\theta$ ← Now, think

$= \frac{\sin^2\theta}{\cos\theta} + \cos\theta \cdot \frac{\cos\theta}{\cos\theta}$ ← LCD is $\cos\theta$

$= \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$ ✓