I. Cont.

eg. Write $\frac{1+\cot^2\theta}{1-\csc^2\theta}$ in terms of $\sin\theta$ and $\cos\theta$, and simplify the expression so that no quotient would appear.

sol:

$$= \frac{1+\left(\frac{\cos\theta}{\sin\theta}\right)^2}{1-\left(\frac{1}{\sin\theta}\right)^2}$$

if you forget: $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$

$$= \frac{1+\frac{\cos^2\theta}{\sin^2\theta}}{1-\frac{1}{\sin^2\theta}}$$

add/subtract fraction

$2. \frac{\sin^2\theta}{\sin^2\theta}$

$$= \frac{\frac{\sin^2\theta}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta}{\sin^2\theta}-\frac{1}{\sin^2\theta}}$$

$$= \frac{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}{\frac{\sin^2\theta-1}{\sin^2\theta}}$$

connect to $\sin^2\theta+\cos^2\theta=1$

connect to

$$\begin{aligned} \sin^2\theta+\cos^2\theta &= 1 \\ -1 \qquad\qquad & \quad -1 \\ \sin^2\theta-1+\cos^2\theta &= 0 \\ -\cos^2\theta \quad & -\cos^2\theta \\ \sin^2\theta-1 &= -\cos^2\theta \end{aligned}$$

$$= \frac{\frac{1}{\sin^2\theta}}{\frac{-\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{1}{\cancel{\sin^2\theta}}\cdot\frac{\cancel{\sin^2\theta}}{-\cos^2\theta}$$

$$= -\frac{1}{\cos^2\theta}$$

$-\left(\frac{1}{\cos\theta}\right)^2$

$$= \boxed{-\sec^2\theta}$$

Eg. Create an identity for the expression $2\tan\theta\sec\theta$ by rewriting strictly in terms of sine.

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Sol:

$$= 2 \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$

$$= 2 \cdot \frac{\sin\theta}{\cos^2\theta}$$

connect to: $\sin^2\theta + \cos^2\theta = 1$

$-\sin^2\theta \qquad -\sin^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$$= \boxed{\frac{2\sin\theta}{1-\sin^2\theta}}$$

Eg. Verify the identity: $(1 - \cos^2 x)(1 + \cot^2 x) = 1.$

Sol: $\sin^2 x(1 + \cot^2 x)$

$= \sin^2 x \cdot \csc^2 x$

$= \sin^2 x \cdot \left(\frac{1}{\sin x}\right)^2$

$= \cancel{\sin^2 x}^{\,1} \cdot \frac{1}{\cancel{\sin^2 x}_{\,1}}$

$= 1$ ✓

$\sin^2\theta + \cos^2\theta = 1$

$-\cos^2\theta \qquad -\cos^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

Remember: $\tan^2\theta + 1 = \sec^2\theta$

Then, $\cot^2\theta + 1 = \csc^2\theta$

Eg. Verify the identity below:

$$\frac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos\theta - \sin\theta$$

Sol:

$$= \frac{(\sin(-\theta))^2 - (\cos(-\theta))^2}{-\sin\theta - \cos\theta}$$

$$= \frac{(-\sin\theta)^2 - (\cos\theta)^2}{\ldots}$$

sine fct is odd: $\sin(-\theta) = -\sin\theta$

cosine fct is even: $\cos(-\theta) = \cos\theta$

$\sin^2(-\theta) = (\sin(-\theta))^2$

$$= \frac{(-\sin\theta)^2 - (\cos\theta)^2}{-\sin\theta - \cos\theta}$$

$\sin^2(-\theta) = (\sin(-\theta))^2$

$\cos^2(-\theta) = (\cos(-\theta))^2$

$$= \frac{\sin^2\theta - \cos^2\theta}{-\sin\theta - \cos\theta}$$

Connect to: $a^2 - b^2 = (a+b)(a-b)$

$$= \frac{\cancel{(\sin\theta + \cos\theta)}^{\,1}(\sin\theta - \cos\theta)}{-1\cdot\cancel{(\sin\theta + \cos\theta)}_{\,1}}$$

factor -1 to prepare $(\sin\theta + \cos\theta)$

$$= -(\sin\theta - \cos\theta)$$

$$= -\sin\theta + \cos\theta$$

$$= \cos\theta - \sin\theta \quad \checkmark$$