Eg. Verify the identity $\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

Sol: $\sec^2\theta - 1$ connects to $\tan^2\theta + 1 = \sec^2\theta$

$$-1 \qquad -1$$

$$\tan^2\theta = \sec^2\theta - 1 \checkmark$$

$$\frac{\sec^2\theta - 1}{\sec^2\theta} = \frac{\tan^2\theta}{\sec^2\theta}$$

$$= \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$$

$$= \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \frac{\cancel{\cos^2\theta}}{1}$$

$$= \sin^2\theta \checkmark$$

Another way: $\frac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

Sol: connects to: $\frac{t-1}{t} = \frac{t}{t} - \frac{1}{t}$

$$\frac{\sec^2\theta - 1}{\sec^2\theta} = \frac{\cancel{\sec^2\theta}^{\,1}}{\cancel{\sec^2\theta}_{\,1}} - \frac{1}{\sec^2\theta}$$

$$= 1 - \frac{1}{\sec^2\theta}$$

$$= 1 - \frac{1}{\frac{1}{\cos^2\theta}} \leftarrow 1 \cdot \frac{\cos^2\theta}{1}$$

$$= 1 - \cos^2\theta \leftarrow \text{Connects to } \sin^2\theta + \cos^2\theta = 1$$

$= 1 - \cos^2\theta$ $\leftarrow$ Connects to $\sin^2\theta + \cos^2\theta = 1$

$\quad -\cos^2\theta \quad -\cos^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$= \sin^2\theta$ ✓

Eg.

Write the following trigonometric expression as an algebraic expression: $2\cos^2\theta + \cos\theta - 1$.

Sol: "$2u^2 + u - 1$", $u = \cos\theta$

or "$2x^2 + x - 1$", $x = \cos\theta$

$2 \cdot -1 = -2$

$2 + -1 = 1$ ✓

$2x^2 + 2x - x - 1$

$(2x^2 + 2x) + (-x - 1)$

$2x(x+1) + -1(x+1)$

$(2x-1)(x+1)$

Thus, $\boxed{(2\cos\theta - 1)(\cos\theta + 1)}$

Eg.

Rewrite the trigonometric expression using the difference of squares: $4\cos^2\theta - 1$.

Sol: Difference of Squares: $a^2 - b^2 = (a+b)(a-b)$

$x = \cos\theta$

$4x^2 - 1$

$= "a^2 - b^2"$

$= (2x)^2 - 1^2$ $\leftarrow$ $4x^2 = (\ \ )^2$

$4x^2 = 2^2 \cdot x^2$

$4x^2 = (2x)^2$

$= ((2x)+1)((2x)-1)$

$= (2x+1)(2x-1)$

Perfect-Square Trinomial:

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$= (2x+1)(2x-1)$

Thus, $\boxed{(2\cos\theta + 1)(2\cos\theta - 1)}$

eg. $25 - 9\sin^2\theta$

$(\sin\theta)^2$

Sol: $25 = 5^2$, $9\sin^2\theta = 3^2 \cdot \sin^2\theta = (3\sin\theta)^2$

$= 5^2 - (3\sin\theta)^2 \leftarrow a^2 - b^2$

$= \boxed{(5 + 3\sin\theta)(5 - 3\sin\theta)}$

$\boxed{3-8 \neq 8-3}$

eg. $8\cos^2\theta - 18\sin^2\theta$.

$= 2(4\cos^2\theta - 9\sin^2\theta) \leftarrow$ 8, 18 is even

$= 2\left((2\cos\theta)^2 - (3\sin\theta)^2\right)$

$= 2\left((2\cos\theta + 3\sin\theta)(2\cos\theta - 3\sin\theta)\right)$

$= \boxed{2(2\cos\theta + 3\sin\theta)(2\cos\theta - 3\sin\theta)}$

eg. $16\tan^2\theta - 24\tan\theta + 9 \leftarrow a^2 - 2ab + b^2$

$= (\overset{a}{4\tan\theta})^2 - \,? + \overset{b}{3}^2$

expect $2ab$

$= (4\tan\theta)^2 - 2\cdot 4\tan\theta \cdot 3 + 3^2$

$2 \cdot 4\tan\theta \cdot 3 = 24\tan\theta$

$= \boxed{(4\tan\theta - 3)^2}$

$$= \boxed{(4\tan\theta - 3)^2}$$