III. Double-Angle Identity

important

$$\boxed{\begin{aligned} \cos 2A &= \begin{cases} \cos^2 A - \sin^2 A \\ 2\cos^2 A - 1 \\ 1 - 2\sin^2 A \end{cases} \\ \sin 2A &= 2\sin A \cos A \\ \tan 2A &= \frac{2\tan A}{1-\tan^2 A} \end{aligned}}$$

$$\tan(A+A) = \frac{\tan A + \tan A}{1 + \tan A \tan A}$$

$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

Pf: $\cos 2A = \cos(A+A)$

$= \cos A \cos A - \sin A \sin A$

$= \cos^2 A - \sin^2 A$ ✓

$\cos 2A = \underline{\cos^2 A} - \sin^2 A$

$= (1 - \sin^2 A) - \sin^2 A$

$= 1 - \sin^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$ ✓

$\cos 2A = \cos^2 A - \sin^2 A$

$= \cos^2 A - (1 - \cos^2 A)$

$= \cos^2 A - 1 + \cos^2 A$

$= 2\cos^2 A - 1$ ✓

⋮

✓

eg. Simplify $\cos^2 7x - \sin^2 7x$

sol: $= \cos(2(7x))$

$= \boxed{\cos 14x}$

eg. Simplify $\sin 15° \cos 15°$ ⟵ recognize $\sin 2A = 2\sin A \cos A$

Sol: We have $\sin 2\theta = 2\sin\theta\cos\theta$,

then $\sin(2(15°)) = 2\sin 15° \cos 15°$

$\frac{\sin 30°}{2} = \frac{\cancel{2}^{1} \sin 15° \cos 15°}{\cancel{2}_{1}}$ ⟵ Want $\sin 15° \cos 15°$

$\frac{\frac{1}{2}}{2} = \sin 15° \cos 15°$

$\boxed{\frac{1}{4}} = \sin 15° \cos 15°$

Another way:

eg. Simplify $\sin 15° \cos 15°$ ⟵ recognize $\sin 2A = 2\sin A \cos A$

sol: $\sin 15° \cos 15° = \underbrace{\frac{1}{2} \cdot 2}_{1} \sin 15° \cos 15°$

$= \frac{1}{2} \cdot 2\sin 15° \cos 15°$

$= \frac{1}{2}\sin(2(15^\circ))$

$= \frac{1}{2}\sin 30^\circ$

$= \frac{1}{2}\cdot\frac{1}{2}$

$= \boxed{\frac{1}{4}}$

eg. Evaluate $5\sin\frac{\pi}{8}\cos\frac{\pi}{8}$.

Sol:

recognize $\sin 2A = 2\sin A\cos A$

We have $\sin 2A = 2\sin A\cos A$,

then $\frac{\sin 2(\frac{\pi}{8})}{2} = \frac{\cancel{2}^{1}\sin\frac{\pi}{8}\cos\frac{\pi}{8}}{\cancel{2}_{1}}$

$\frac{\sin 2(\frac{\pi}{8})}{2} = \sin\frac{\pi}{8}\cos\frac{\pi}{8}$

what about '5'?

$\boxed{\cancel{2}^{1}\cdot\frac{\pi}{\cancel{8}_4} = \frac{\pi}{4}}$

$5\cdot(\qquad) = 5\cdot(\qquad)$

$5\left(\frac{\sin 2(\frac{\pi}{8})}{2}\right) = 5\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$5\left(\frac{\sin\frac{\pi}{4}}{2}\right) =$

$5\left(\frac{\frac{\sqrt{2}}{2}}{2}\right) =$ $\qquad 5\cdot\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$\boxed{\frac{5\sqrt{2}}{4}} =$

<u>Different way:</u>

eg. Evaluate $5\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

eg. Evaluate $5\sin\frac{\pi}{8}\cos\frac{\pi}{8}$.

recognize $\sin 2A = 2\sin A\cos A$

Sol: $= 5\cdot\underbrace{\frac{1}{2}\cdot 2}_{1}\cdot\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= 5\cdot\frac{1}{2}\cdot 2\sin\frac{\pi}{8}\cos\frac{\pi}{8}$

$= 5\cdot\frac{1}{2}\sin\left(2\left(\frac{\pi}{8}\right)\right)$

$= 5\cdot\frac{1}{2}\cdot\sin\frac{\pi}{4}$

$= 5\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \boxed{\frac{5\sqrt{2}}{4}}$

eg. Evaluate $-3\sin 22.5^\circ\cos 22.5^\circ$.

Sol: $= -3\cdot\frac{1}{2}\cdot 2\cdot\sin 22.5^\circ\cos 22.5^\circ$

$= -3\cdot\frac{1}{2}\cdot 2\sin 22.5^\circ\cos 22.5^\circ$

$= -3\cdot\frac{1}{2}\cdot\sin(2(22.5^\circ))$

$= -3\cdot\frac{1}{2}\cdot\sin 45^\circ$

$= -3\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \boxed{-\frac{3\sqrt{2}}{4}}$

eg. Simplify $4\cos^2 5k - 2$.

connect to

5. Simplify $4\cos^2 5k - 2$.

connect to $2\cos^2 A - 1$

Sol: $= 2(2\cos^2 5k - 1)$

$= 2\cos(2(5k))$

$= \boxed{2\cos 10k}$