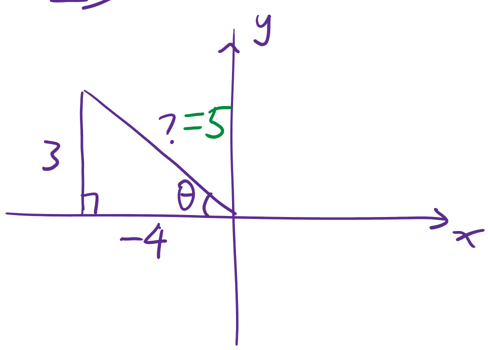
III. Cont.

Eg. Using a Double-Angle Formula to Find the Exact Value Involving Tangent

Given that $\tan\,\theta = -\frac{3}{4}$ and θ is in quadrant II, find the following:

ⓐ $\sin(2\theta)$ ⓑ $\cos(2\theta)$ ⓒ $\tan(2\theta)$

Sol: $\tan\theta$ in QII $\Rightarrow$



Then,
$$3^2 + (-4)^2 = ?^2$$
$$9 + 16 = ?^2$$
$$\sqrt{\ } \; 25 = ?^2 \; \sqrt{\ }$$
$$\pm 5 = ?$$
$$5 = ?$$

ⓐ
$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$= 2 \cdot \frac{3}{5} \cdot -\frac{4}{5}$$
$$= \boxed{-\frac{24}{25}}$$

ⓑ
$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$= 2 \cdot \left(-\frac{4}{5}\right)^2 - 1$$
$$= 2 \cdot \frac{16}{25} - 1$$
$$= \frac{32}{25} - 1 \cdot \frac{25}{25}$$
$$= \boxed{\frac{7}{25}}$$

ⓒ
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$
$$= \frac{\cancel{2} \cdot \left(-\frac{3}{\cancel{4}_2}\right)}{1 - \left(-\frac{3}{4}\right)^2}$$
$$= \frac{-\frac{3}{2}}{1 - \frac{9}{16}}$$
$$= \frac{-\frac{3}{2}}{\frac{7}{16}}$$
$$= -\frac{3}{\cancel{2}_1} \cdot \frac{\cancel{16}^8}{7}$$

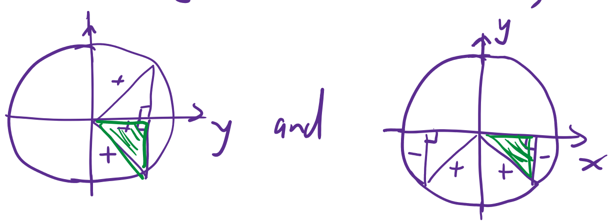
$$1 - \frac{9}{16}$$
$$\frac{16}{16} - \frac{9}{16}$$

$$= -\frac{3}{1\cancel{2}}\cdot\frac{\cancel{16}8}{7}$$

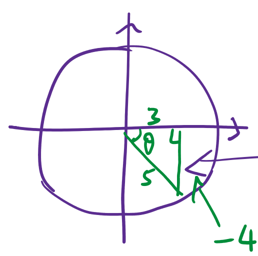
$$= \boxed{-\frac{24}{7}}$$

eg. If $\cos\theta = \frac{3}{5}$ and $\sin\theta < 0$. Find the exact value of $\cos(2\theta)$.

Sol: For $\cos\theta = \frac{3}{5}$ and $\sin\theta < 0$, then



$\Rightarrow$



$3^2 + ?^2 = 5^2$

$9 + ?^2 = 25$

$-9 \qquad -9$

$\sqrt{}\; ?^2 = 16\; \sqrt{}$

$? = \pm 4$

$? = -4 \leftarrow$ it's y in Q4.

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$= 1 - 2\cdot\left(-\frac{4}{5}\right)^2$$

$$= 1 - 2\cdot\frac{16}{25}$$

$$= 1 - \frac{32}{25}$$

$$= \boxed{-\frac{7}{25}}$$

$$1\cdot\frac{25}{25} - \frac{32}{25}$$

Eg.

$(a+b)^2 = a^2 + 2ab + b^2$

Using the Double-Angle Formulas to Verify an Identity

Verify the following identity using double-angle formulas:

$$1 + \sin(2\theta) = (\sin\ \theta + \cos\ \theta)^2$$

trick : $\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

Sol: $1 + \sin(2\theta) = 1 + 2\sin\theta\cos\theta$ ← Can't go further

Now, $(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta$

$= 1 + \sin(2\theta)$ ✓

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

Eg.

Verifying a Double-Angle Identity for Tangent

Verify the identity:

$$\tan(2\theta) = \frac{2}{\cot\ \theta - \tan\ \theta}$$

Sol: $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$= \dfrac{2\tan\theta \,/\tan\theta}{1-\tan^2\theta\,/\tan\theta}$ → $\dfrac{1-\tan^2\theta}{\tan\theta}$ " $\dfrac{5-x^2}{x}$ " $\dfrac{5}{x} - \dfrac{x^2}{x}$

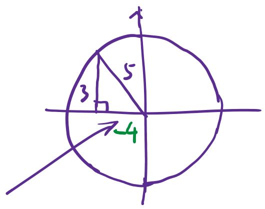
$= \dfrac{\frac{2\cancel{\tan\theta}}{\cancel{\tan\theta}}}{\frac{1}{\tan\theta} - \frac{\cancel{\tan^2\theta}}{\cancel{\tan\theta}}}$

$= \dfrac{2}{\cot\theta - \tan\theta}$ ✓

$$= \frac{2}{\cot\theta - \tan\theta} \checkmark$$

eg. If $\sin\theta = \frac{3}{5}$, $\frac{\pi}{2} < \theta < \pi$. Find $\sin 2\theta$.

Sol:



$$3^2 + x^2 = 5^2$$

$$9 + x^2 = 25$$

$$-9 \qquad -9$$

$$x^2 = 16$$

$$x = \pm 4$$

$x = -4$ because x in Q2.

Then, $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2 \cdot \frac{3}{5} \cdot -\frac{4}{5}$$

$$= \boxed{-\frac{24}{25}}$$