V. Other Trig Properties ← not important (won't be tested)

It is good to learn for excelling in competition.

Reduction:

$$\sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$\tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Addition:

The **product-to-sum formulas** are as follows:

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta)+\sin(\alpha-\beta)]$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta)-\cos(\alpha+\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta)-\sin(\alpha-\beta)]$$

The **sum-to-product formulas** are as follows:

$$\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha-\sin\beta = 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

$$\cos\alpha-\cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha+\cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

↑

Remember them, if you really want to!

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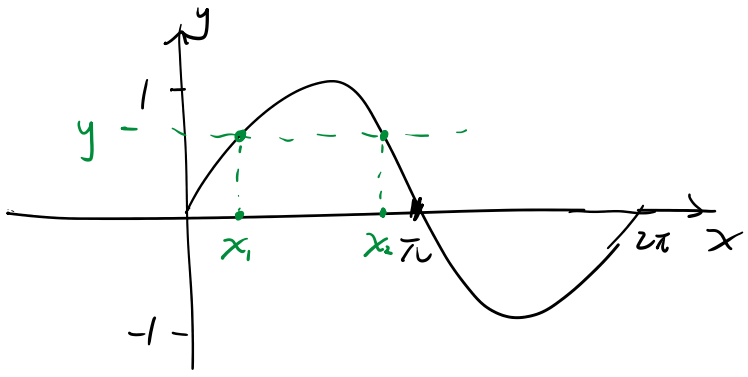
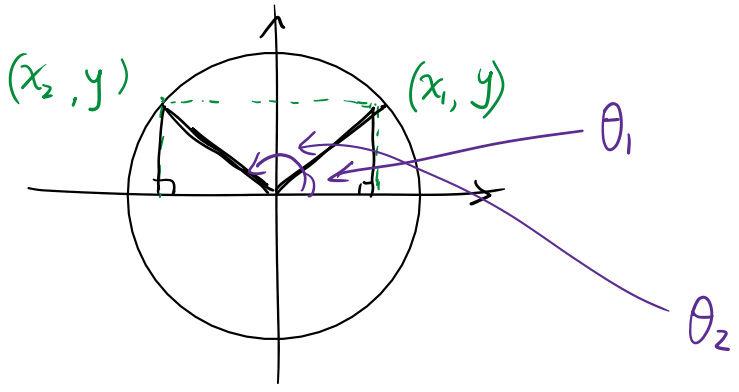
VI. Solve Trigonometric Equation

eg.



Thus, x_1 and x_2 are solution, since they produced the same y.

For



$(x, y) = (\cos\theta, \sin\theta)$

$(*)^2 + 2(*) = 0$

$\sin^2\theta + 2\sin\theta = 0$

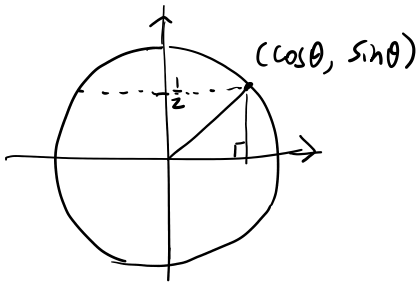
Steps: 1. Pretend the trig fct as a variable x, y ...

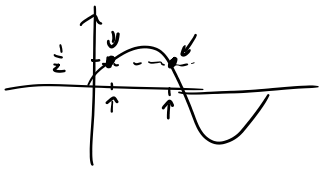
2. Solve the equation regularly, with algebraic operation.

3. Check the period, and solve again. (Within the period)

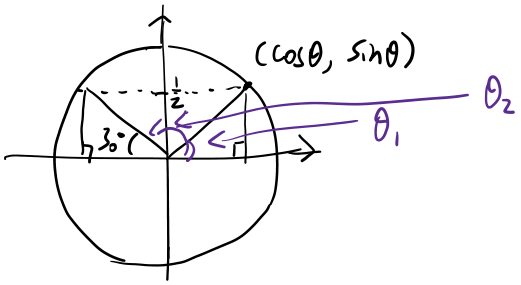
eg. Solve $\sin\theta = \frac{1}{2}$ in $0 \leq \theta \leq 2\pi$.

Sol:





$\Rightarrow$



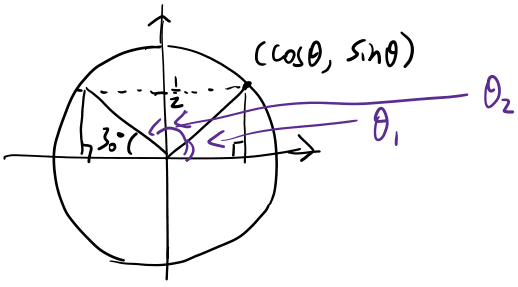
$\sin^{-1}\sin\theta = \sin^{-1}\frac{1}{2}$ ← $180°-30°$

$\theta = 30°, 150°$

$\theta_1 = \boxed{30°}$, $\theta_2 = \boxed{150°}$ $\left(\theta_1 = \boxed{\frac{\pi}{6}}, \theta_2 = \boxed{\frac{5\pi}{6}}\right)$

eg. Solve $\sin\theta = \frac{1}{2}$ in general. ← any period

Sol:



$\sin^{-1}\sin\theta = \sin^{-1}\frac{1}{2}$ ← $180°-30°$

$\theta = 30°, 150°$

$\theta_1 = 30°$, $\theta_2 = 150°$ ← for one period only, not general

General: $\theta_1 = \boxed{30° + 360°n}$, $\theta_2 = \boxed{150° + 360°n}$

n is a natural

$= \boxed{30°(1+12n)}$ $= \boxed{30°(5+12n)}$, n is a natural number

(or $\theta_1 = \boxed{\frac{\pi}{6} + 2\pi n}$, $\theta_2 = \boxed{\frac{5\pi}{6} + 2\pi n}$)

eg. Solve the equation $3\sin x - 2 = 5\sin x - 1$.

doesn't specify from 0 to 2π. It means for general

Sol: $3\sin x - 2 = 5\sin x - 1$

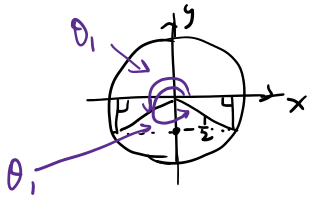
$-5\sin x \qquad -5\sin x$

$-2\sin x - 2 = -1$

$+2 \qquad +2$

$\frac{-2\sin x}{-2} = \frac{1}{-2}$

$\sin x = -\frac{1}{2}$



$(\cos\theta, \sin\theta)$

$\uparrow$

$\leftarrow -\frac{1}{2}$

$\theta_1 = \pi + \frac{\pi}{6}$, $\theta_2 = 2\pi - \frac{\pi}{6}$

$= \frac{7\pi}{6}$ $= \frac{11\pi}{6}$

$2\pi - \frac{\pi}{6}$

$= 2\pi \cdot \frac{6}{6} - \frac{\pi}{6}$

$= \frac{12\pi}{6} - \frac{\pi}{6}$

$= \frac{11\pi}{6}$

not general

General: $\theta_1 = \boxed{\frac{7\pi}{6} + 2n\pi}$, $\theta_2 = \boxed{\frac{11\pi}{6} + 2n\pi}$ for n is a natural number.

eg. Solve $\sqrt{3}\tan\theta + 1 = 0$.

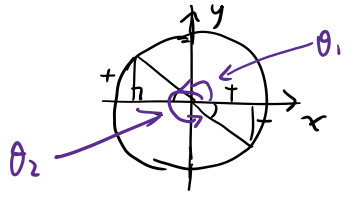
Sol:

$$\sqrt{3}\tan\theta + 1 = 0$$
$$-1 \quad -1$$

$$\frac{\sqrt{3}\tan\theta}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$$

$$\tan\theta = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan\theta = -\frac{\sqrt{3}}{3}$$



$\tan\theta$ is $\frac{x}{y}$

$\leftarrow$ period for $\tan\theta$: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

π

$\theta_1 = \pi - \frac{\pi}{6}$, $\theta_2 = 2\pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$, $= \frac{11\pi}{6}$

$\theta_1 = \frac{5\pi}{6} + n\pi$, $\theta_2 = \frac{11\pi}{6} + n\pi$

$\Rightarrow$ $\theta = \boxed{\frac{5\pi}{6} + n\pi}$ $\leftarrow$ θ_1 includes θ_2