VI. Cont.

Eg.

Solving a Trigonometric Equation Using Algebra

Solve exactly:

tricky "<", not $\leq$

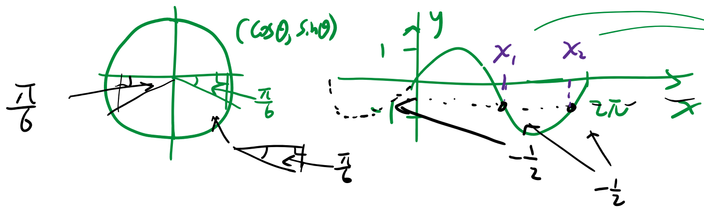
$$2\sin^2\theta + \sin\theta = 0;\ 0 \leq \theta < 2\pi$$

"$2x^2 + x = 0$"

Sol: $\sin\theta(2\sin\theta + 1) = 0$

$\underbrace{\sin\theta}\cdot\underbrace{(2\sin\theta+1)} = 0$

$\sin\theta = 0, \quad (2\sin\theta + 1) = 0$



$\theta = 0, \pi \quad , \qquad 2\sin\theta + 1 = 0$

$\qquad\qquad\qquad\qquad -1 \quad -1$

$\frac{\cancel{2}\sin\theta}{\cancel{2}} = \frac{-1}{2}$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{6} + \pi, \quad 2\pi - \frac{\pi}{6}$

$\theta = \frac{7\pi}{6}, \ \frac{11\pi}{6}$

$\frac{\pi}{6} + \pi = \frac{\pi}{6} + \frac{6\pi}{6} = \frac{7\pi}{6}$

$2\pi - \frac{\pi}{6} = 2\pi\cdot\frac{6}{6} - \frac{\pi}{6} = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$

For $0 \leq \theta < 2\pi$, $\theta = \boxed{0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}}$

eg. Solve $\sqrt{2}\cos(2\theta) + 1 = 0$.

no restriction, it asks for the general solutions

Sol. $\sqrt{2}\cos(2\theta) + 1 = 0$

$\qquad -1 \quad -1$

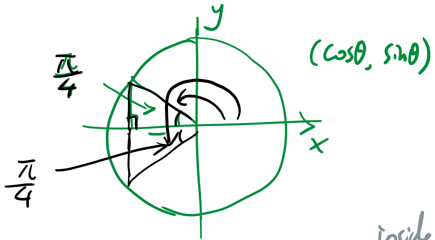
2θ is inside cos

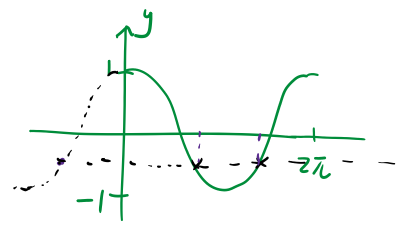
$\frac{\cancel{\sqrt{2}}\cos(2\theta)}{} = \frac{-1}{}$

$$\frac{\cancel{\sqrt{2}}\cos(2\theta)}{\cancel{\sqrt{2}}} = \frac{-1}{\sqrt{2}}$$

$$\cos(2\theta) = -\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$\cos(2\theta) = -\frac{\sqrt{2}}{2}$$





$$\pi - \frac{\pi}{4} = \frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$$

inside cos | inside cos

$2\theta = \pi - \frac{\pi}{4}$, $\quad 2\theta = \pi + \frac{\pi}{4}$

$2\theta = \frac{3\pi}{4}$, $\quad 2\theta = \frac{5\pi}{4}$

$\frac{\cancel{2}\theta}{\cancel{2}} = \frac{\frac{3\pi}{4} + 2n\pi}{2}$, $\quad \frac{\cancel{2}\theta}{\cancel{2}} = \frac{\frac{5\pi}{4} + 2n\pi}{2}$

$\leftarrow \frac{3\pi}{4}\cdot\frac{1}{2}$

$\leftarrow \frac{5\pi}{4}\cdot\frac{1}{2}$

$\theta = \frac{3\pi}{8} + n\pi$, $\quad \theta = \frac{5\pi}{8} + n\pi$

$\theta = \boxed{\frac{3\pi}{8} + n\pi, \quad \frac{5\pi}{8} + n\pi}$, where n is an integer.

eg. Consider the equation $\sqrt{3}\tan\left(\frac{\theta}{2}\right) - 1 = 0$.

a. Find the general solutions. $\leftarrow$ $\frac{\theta}{2}$ inside tan

b. Find all solutions in $0 \le \theta < 4\pi$.

Sol: a. $\sqrt{3}\tan\left(\frac{\theta}{2}\right) - 1 = 0$

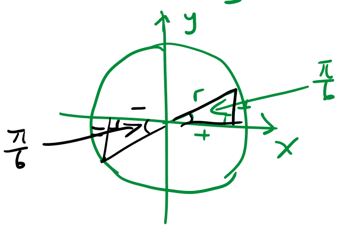
$\qquad\qquad +1 \quad +1$

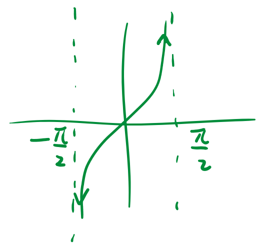
$$\frac{\cancel{\sqrt{3}}\tan\left(\frac{\theta}{2}\right)}{\cancel{\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

$\tan\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan\left(\frac{\theta}{2}\right) = \frac{\sqrt{3}}{3}$





$\frac{\theta}{2} = \frac{\pi}{6}$, $\quad \frac{\theta}{2} = \pi + \frac{\pi}{6}$

$\pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6}$
$= \frac{7\pi}{6}$

$\frac{\theta}{2} = \frac{\pi}{6}$, $\quad \frac{\theta}{2} = \frac{7\pi}{6}$

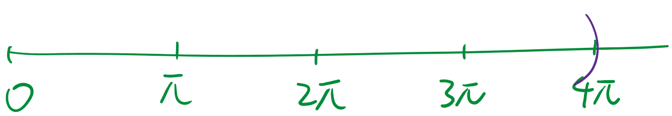
$2 \cdot \frac{\theta}{\cancel{2}} = \left(\frac{\pi}{6} + n\pi\right) \cdot 2$, $\quad 2 \cdot \frac{\theta}{\cancel{2}} = \left(\frac{7\pi}{6} + n\pi\right) \cdot 2$

$\theta = \frac{\pi}{3} + 2n\pi$, $\quad \frac{7\pi}{6}$ is not in $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\cancel{\theta = \left(\frac{7\pi}{6} + n\pi\right) \cdot 2}$

$\theta = \boxed{\frac{\pi}{3} + 2n\pi}$, n is an integer

b. $0 \le \theta < 4\pi$



$4\pi \cdot \frac{3}{3} = \frac{12\pi}{3}$

$n = 0$,
$\theta = \frac{\pi}{3} + 2 \cdot 0 \cdot \pi$
$= \frac{\pi}{3} + 0$
$= \frac{\pi}{3}$

$n = 1$,
$\theta = \frac{\pi}{3} + 2 \cdot 1 \cdot \pi$
$= \frac{\pi}{3} + 2\pi$
$= \frac{7\pi}{3}$

$n = 2$,
$\theta = \frac{\pi}{3} + 2 \cdot 2 \cdot \pi$
$= \frac{\pi}{3} + 4\pi$
$= \cancel{\frac{13\pi}{3}} > \frac{12\pi}{3}$

$\frac{\pi}{3} + 2\pi$
$= \frac{\pi}{3} + \frac{6\pi}{3}$

$= \frac{\pi}{3}$ $= \frac{7\pi}{3}$ $= \cancel{\frac{13\pi}{3}}$ $> \frac{12\pi}{3}$)

$\theta = \boxed{\frac{\pi}{3}, \frac{7\pi}{3}}$